## Advanced Programming Languages (COP 4930/CIS 6930) [Spring 2015]

## Assignment V

**Due Date:** Monday 3/23/15 at 5pm

## **Assignment Description**

Do the following by yourself (please don't discuss solutions until after the due date).

Consider a language L with the following types.

 $\tau ::= \bot \mid \top \mid \{l_1:\tau_1 \dots l_n:\tau_n\} \mid \tau_1 - > \tau_2 \mid \mu t.\tau \mid t$ 

Assume that all types under consideration start out with no free (type) variables, alphaconversion for type variables has already been defined, and all types under consideration start out containing "uniquified" type variables.

a) Define the subtyping relation for L using deterministic (algorithmic) rules.

b) Define a join relation (least upper bound) for types in L, again deterministically.

For a concrete example of joins, let  $\tau_1$  be  $\bot - > \bot$  and  $\tau_2$  be  $\top - > \top$ . Neither  $\tau_1$  nor  $\tau_2$  is a subtype of the other. Nonetheless, we can compute the join of  $\tau_1$  and  $\tau_2$  as  $\bot - > \top$ . If the "then" branch of an if-then-else expression has type  $\tau_1$ , and the "else" branch has type  $\tau_2$ , then  $\bot - > \top$  could be used as the type of the whole conditional expression because both  $\tau_1$  and  $\tau_2$  are subtypes of  $\bot - > \top$ , and no other type more precisely characterizes the types of both branches.