A Calculus for Composing Security Policies*

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Abstract

A runtime monitor is a program that runs in parallel with an untrusted application and examines actions from the application's instruction stream. If the sequence of program actions deviates from a specified security policy, the monitor transforms the sequence or terminates the program. We present the design and formal specification of a language for defining the policies enforced by program monitors.

Our language provides a number of facilities for composing complex policies from simpler ones. We allow policies to be parameterized by values, or other policies. There are also operators for forming the conjunction and disjunction of policies. Since the computations that implement these policies modify program behavior, naive composition of computations does not necessarily produce the conjunction (or disjunction) of the policies that the computations implement separately. We use a type and effect system to ensure that computations do not interfere with one another when they are composed. We also present a preliminary implementation of our language.

1 Introduction

Any system designed to execute and interoperate with potentially malicious code should implement at least two different sorts of security mechanisms:

1. A safe language and sound type checker to statically rule out simple bugs.
2. A run-time environment to detect, document, prevent and recover from those errors that cannot be detected beforehand.

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Strong type systems such as the ones in the Java Virtual Machine [LY99] and Common Language Runtime [GS01, Gou01, MG] are the most efficient and most widely deployed mechanisms for ruling out a wide variety of potential security holes ranging from buffer overruns to misuse of system interfaces.

To complement static checking, secure run-time environments normally use auxiliary mechanisms to check properties that cannot be decided at compile time or link time. One of the ways to implement such run-time checks is with program monitors, which examine a sequence of program actions before they are executed. If the sequence deviates from a specified policy, the program monitor transforms the sequence or terminates the program.

In this paper, we describe a new general-purpose language called Polymer that can help designers of secure systems detect, prevent and recover from errors in untrusted code at runtime. System architects can use Polymer to write program monitors that run in parallel with an untrusted application. Whenever the untrusted application is about to call a security-sensitive method, control jumps to the Polymer program which determines which of the following will occur:

- the application runs the method and continues with its computation,
- the application is terminated by the monitor,
- the application is not allowed to invoke the given method, but otherwise may continue with its computation, or
- the monitor performs some computation on behalf of the application before or after proceeding with any of the first three options (Figure 1).

This basic architecture has been used before to implement secure systems [DG71, ES00, ET99, GB01, PH00]. Previous work has shown that the framework effectively subsumes a variety of less general mechanisms such as access-control lists and stack inspection. Unfortunately, there has been a nearly universal lack of concern for precise semantics for these languages, which we seek to remedy in this paper.

We improve upon previous work in a number of ways.

- We present a new, general-purpose language for designing run-time security policies. Policies are able to prevent actions from being executed, execute their own actions, and terminate the offending program. In our language, policies are first-class objects and can be parameterized by other policies or ordinary values. We provide interesting and useful combinators that allow complex policies to be built from simpler ones.
- We define a formal operational semantics for our language, which turns out to be a variant of the computational lambda calculus [Mog91]. To our knowledge, this is the first such semantics for a general-purpose security monitoring language. It provides a tool that system architects can use to reason precisely about their security policies.

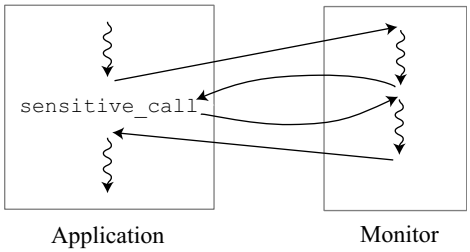

Figure 1: Sample interaction between application and monitor: monitor allows application to make a sensitive call.

- We provide a type system, which we have proven sound with respect to our operational semantics. The type system includes a novel effect system that ensures that composed policies do not interfere with one another.
- We have developed a preliminary implementation of our language that enforces policies on Java programs.

2 A Calculus for Composing Security Policies

In this section, we provide an informal introduction to our security policy language. The next section will define a rigorous semantics for the language.

2.1 Simple Policies

Our monitoring language is derived from Moggi's computational lambda calculus [Mog91]; consequently, the language constructs are divided into two groups: pure terms M and computations E. A computation runs in parallel with a *target program* and may have effects on the target's behavior. We call a suspended computation paired with an action set ($\{\texttt{actions} : A; \texttt{policy} : E\}$) a *policy*. A policy is a term that, when its suspended computation E is run, will intercept and manipulate *target actions* in the set A. We call this set of actions the *regulated set*. For the purposes of this paper, a target action is a function or method call that the target application wishes to execute. However, it is easy to imagine a variety of other sorts of target program actions, such as primitive operations like assignment, dereference, iteration, the act of raising particular exceptions, etc., that might also be considered actions that are regulated by a security policy.

A first example Consider the following policy, which enforces a limit on the amount of memory that an application can allocate for itself.

```
fun mpol(q:int).
{
  actions:  malloc();
  policy:
    next →
      case ⋆ of
        malloc(n) →
          if ((q-n) > 0) then
            ok; run (mpol (q-n))
          else
            halt
      end
    done → ()
}
```

A recursive policy, like the one above, is a recursive function (a term) with a policy as its body. The recursive function argument `q` is a memory quota that the application must not exceed. The only action manipulated by this policy is the `malloc` action. The computation defining the policy begins with the (`next` $\rightarrow E_1 \mid$ `done` $\rightarrow E_2$) computation, which suspends the monitor until the target is about to execute the next action in the regulated set (i.e., the next malloc operation). At this point, the monitor executes E_1. If the program terminates before before executing another regulated action, E_2 will be executed to perform any sort of bookkeeping or application cleanup that is necessary. In this example, we assume no bookkeeping is necessary so the monitor simply returns `()` to indicate it is done.

The `ok` statement signals that the current action should be accepted, and `halt` is the terminal computation, which halts the target program.

A recursive call to a policy involves two steps. The first step is a function application (`mpol (q-n)`), which returns a policy (a suspended computation). To *run* the suspended computation, we use the `run` statement (`run (mpol (q-n))`). Sometimes, computations return interesting values (not just unit) in which case we write `let {x} = pol in E`. This is the monadic let, which executes its primary argument `pol`, binds the resulting value to `x` and continues the computation with `E`. We also use an ordinary let where convenient: `let x = M in E`.

Now that we have defined our recursive memory-limit policy, we can initialize it with a quota (`q0`) simply by applying our recursive function.

```
memLimit = mpol q0
```

The type of any policy is $\mathcal{M}(\tau)$ where τ is the type of the value that the underlying computation returns. Hence, the type of `memLimit` is $\mathcal{M}(\mathsf{unit})$.

A second example In this example, we restrict access to files by controlling the actions `fopen` and `fclose`. For simplicity, we assume that `fclose` takes a string argument rather than a file descriptor. The first argument to the policy

is a function (`acl`) that returns true if the target is allowed access to the given file in the given mode. The second argument is a list of files that the application has opened so far. The code below uses a number of list processing functions including cons (::), membership test (`member`), and element delete (`delete`).

```
fun fpol(acl:string->mode->bool, files:file list).
{
  actions:  fopen(), fclose();
  policy:
    let fcloses fs = {... fclose f ...} in
    next →
      case ⋆ of
        fopen(s,m) →
          if (acl s m) then
            ok; run (fpol acl (s::files))
          else
            run (fcloses files); halt
        fclose(s) →
          if (member files s) then
            ok; run (fpol acl (delete files s))
          else
            sup; run (fpol acl files)
      end
    done →
      run (fcloses files)
}
```

The main additional statement of interest in this policy is the `sup` statement. We view an attempt to close a file that that has not been opened by the application a benign error. In this case, we do not terminate the application, we simply *suppress* the action and allow the application to continue (if it is able to do so). In practice, the `sup` expression also throws a security exception that may be caught by the target.

A second point of interest is the fact that our file-system policy is written so that if the target terminates, it will close any files the target has left open. It uses an auxiliary computation `fcloses` to close all the files in the list.

Once again, we must initialize our policy with appropriate arguments.

```
fileAccess = fpol (acl0,[]).
```

2.2 Composing Policies

One of the main novelties of our language is that policies are first-class values. As a result, functions can abstract over policies and policies may be nested inside other policies. Moreover, we provide a variety of combinators that allow programmers to synthesize complex policies from simpler ones.

Parallel Conjunctive Policies A resource-management policy might want to enforce policies on a variety of different sorts of resources, all defined independently of one another. We use the conjunctive combinator $M_1 \wedge M_2$ to create such a policy. For example, the following policy controls both file access and limits memory consumption.

```
RM = fileAccess ∧ memLimit
```

When this policy is run, target actions are streamed to both `fileAccess` and `memLimit`. Actions such as `malloc`, which are not relevant to the `fileAccess` policy, are ignored by it and automatically deemed okay. The same is true of actions that are not relevant to `memLimit`. The two computations may be seen as running in parallel and if either policy decides to halt the target then the target will be stopped.

The result of a parallel conjunctive policy is a pair of values, one value being returned from each of the two computations. Hence, our resource manager has type $\mathcal{M}(\mathsf{unit} \times \mathsf{unit})$.

Closely related to the parallel conjunctive policy is the trivial policy $\top$, which immediately returns (). The trivial policy is the identity for the parallel conjunctive policy. In other words, $M \wedge \top$ accepts exactly the same sequences of program actions as M.

Higher-order Policies Since policies are ordinary values, we can parameterize policies by other policies. For example, rather than fix a particular resource management policy once and for all, a system designer might prefer to design a generic resource manager that is composed of a file-access policy and a memory limit policy.

```
genericRM = λfa:M(unit).λml:M(unit).{let {x} = fa ∧ ml in ()}
```

The generic resource manager above abstracts two policies and returns another policy that runs the two policies in conjunction, discards their results and returns unit. We can apply the generic resource manager to the two policies we created above.

```
strictRM = genericRM fileAccess memLimit
```

However, we might need a different policy for a different application. For instance, for a more trusted application, we might choose not to limit memory, but still control file access. In this case, we use the trivial policy instead of `memLimit`.

```
laxRM = genericRM fileAccess ⊤
```

Parallel Disjunctive Policies A parallel disjunctive policy $M_1 \vee_\tau M_2$ accepts a sequence of operations and returns a result as soon as either M_1 or M_2 would accept the sequence of operations and return. Both policies must agree to halt the target in order to stop it. As in the conjunctive policy, target actions that are not in the regulated set of one of the policies are simply passed over by that policy and implicitly accepted. A disjunctive policy $M_1 \vee_\tau M_2$ has type $\mathcal{M}(\tau)$ when $\tau = \tau_1 + \tau_2$, M_1 has type $\mathcal{M}(\tau_1)$ and M_2 has type $\mathcal{M}(\tau_2)$.

There are several uses for disjunctive policies. At the most basic level, a disjunctive policy can serve to widen an existing policy. For example, suppose we have already implemented a policy for controlling arbitrary, untrusted applications (`untrustedPol`). Later, we might wish to develop a second policy for more trusted applications that authenticate themselves first (`authenticatedPol`). By using disjunction we allow applications either to authenticate themselves and gain further privileges or to use the untrusted policy.

```
widenedPol = untrustedPol ∨τ authenticatedPol
```

It is likely possible to rewrite `untrustedPol` so that it grants extra privileges when a user authenticates himself. However, modular design principles suggest we should leave the code of the initial policy alone and create a separate module (policy) to handle the details of authentication and the extended privileges.

Disjunctive policies also provide a convenient way to create *Chinese wall policies* [BN89]. A Chinese wall policy allows the target to choose from one of many possible policies. However, when one policy is chosen the others become unavailable. For example, when designing a browser policy, we might expect two different sorts of applets. One sort of applet acts like a proxy for a database or service situated across the net. This kind of applet needs few host system resources other than network access. It takes requests from a user and communicates back to the online database. In particular, it has no use for the file system. Another sort of applet performs tasks for the host system and requires access to host data. In order to allow both sorts of applets to run on the host and yet to protect the privacy of host data, we can create a Chinese wall policy which allows either file-system access or network access but not both.

In the code below, we implement this policy. The patterns `File.*` and `Network.*` match all functions in the interface `File` and `Network` respectively. We assume the policies `filePolicy` and `networkPolicy` have been defined earlier.

```
fileNotNetwork =
{
  actions:  File.*, Network.*;
  policy:
    next →
      case ⋆ of
        File.* → run (filePolicy)
        Network.* → halt
      end
    done → ()
}
networkNotFile =
{
  actions:  File.*, Network.*;
  policy:
    next →
      case ⋆ of
        File.* → halt
        Network.* → run (networkPolicy)
      end
    done → ()
}
ChineseWall = fileNotNetwork ∨τ networkNotFile
```

Like conjunction, disjunction has an identity: $\bot$ is the unsatisfiable policy, which halts immediately regardless of any program actions. The policy $M \vee_\tau \bot$ accepts the same sequences of actions as M.

2.3 Interfering Policies

Composition of policies can sometimes lead to policies that are ill-defined or simply wrong. For example, consider the conjunction of two file-system policies, `liberalFilePolicy` and `stricterFilePolicy`. The first policy okays each file-system action while the second policy suppresses some of the file-system actions. What should the result be when one policy suppresses an action and another concurrently allows (and potentially requires) it to occur?

A similar problem would occur if we attempted to compose our original file-system policy `fileAccess` with a logging policy `logPolicy` that stores the sequence of all actions that occur in a system in order to detect suspicious access patterns and to uncover mistakes in a policy. Our original `fileAccess` itself performs certain actions on behalf of the target, including closing target files. If the logging policy operates concurrently with the file-access policy, it cannot detect and log the actions performed by `fileAccess`.

We propose a twofold solution to such problems. First, we use a type and effect system to forbid ill-defined or interfering policies such as the ones considered above. Second, we provide an alternative set of combinators that allow

programmers to explicitly sequence policies rather than having them execute in parallel. This gives programmers necessary flexibility in defining policies.

Types and Effects Our type and effect system gives policies refined types with the form $\mathcal{M}_{A_e}^{A_r}(\tau)$. The set of actions A_r includes all the actions regulated by the policy. The second set A_e specifies the effect of the policy. In other words, it specifies the actions that may be suppressed or initiated on behalf of the program.

These refined types give rise to a new typing rule for parallel conjunctive policies. In the following rule, the context Γ maps variables to their types in the usual way.

$$\frac{\Gamma \vdash M_1 : \mathcal{M}_{A_2}^{A_1}(\tau_1) \qquad \Gamma \vdash M_2 : \mathcal{M}_{A_4}^{A_3}(\tau_2) \qquad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \wedge M_2 : \mathcal{M}_{A_2 \cup A_4}^{A_1 \cup A_3}(\tau_1 \times \tau_2)}$$

The constraint in the rule specifies that the effects of one of the policies must not overlap with the set of actions regulated by the other policy. A similar rule constrains the policies that may be composed using parallel disjunction.

$$\frac{\Gamma \vdash M_1 : \mathcal{M}_{A_2}^{A_1}(\tau_1) \qquad \Gamma \vdash M_2 : \mathcal{M}_{A_4}^{A_3}(\tau_2) \qquad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \vee_{\tau_1+\tau_2} M_2 : \mathcal{M}_{A_3 \cup A_4}^{A_1 \cup A_2}(\tau_1 + \tau_2)}$$

Rules for typing other terms and rules for typing computations are explained in Section 3.

Sequential Combinators Sequential combinators allow programmers to explicitly order the execution of effectful policies that apply to the same set of target actions. The sequential conjunctive policy $M_1 \triangle M_2$ operates as follows. The policy M_1 operates on the target action stream, creating an output stream that may contain new actions that M_1 has injected into the stream and may be missing actions that M_1 has suppressed. The policy M_2 acts as it normally would on the output of M_1. Since this is a conjunctive policy, if either policy decides to terminate the application then the application will be terminated. The sequential disjunctive policy $M_1 \bigtriangledown_\tau M_2$ is similar: M_2 operates on the output of M_1. In this case, however, both M_1 and M_2 must decide to terminate the target in order for the target to be stopped. If one policy signals halt, the disjunction continues to operate as if that policy has no effect on the target.

The typing rules for sequential combinators (shown below) are much more liberal then the typing rules for parallel combinators. By explicit sequencing of operations, the programmer determines how the conflicting decisions should be resolved.

$$\frac{\Gamma \vdash M_1 : \mathcal{M}_{A_2}^{A_1}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}_{A_4}^{A_3}(\tau_2)}{\Gamma \vdash M_1 \triangle M_2 : \mathcal{M}_{A_2 \cup A_4}^{A_1 \cup A_3}(\tau_1 \times \tau_2)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}_{A_2}^{A_1}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}_{A_4}^{A_3}(\tau_2)}{\Gamma \vdash M_1 \bigtriangledown_{\tau_1+\tau_2} M_2 : \mathcal{M}_{A_2 \cup A_4}^{A_1 \cup A_3}(\tau_1 + \tau_2)}$$

Because sequential operators accept a wider range of policies than parallel ones, they can be used to implement any policy that can be implemented with parallel combinators. Parallel combinators, however, ensure the often-desirable property that the two policies being composed do not interfere with each other.

3 Formal Semantics

This section describes the syntax and formal semantics of our calculus. The complete rules for the operational and static semantics are included in Appendices A and B.

3.1 Syntax

The syntax of our formal language differs slightly from the syntax used in the previous section. First, we use the metavariable a to range over actions and consider them to be atomic symbols rather than decomposable into class name, method name and arguments. Second, we write the regulated set for a policy using superscript notation: $\{E\}^A$ is the simple policy with the regulated set A and computation E.

There are also a number of differences in the computations. Our `acase` instruction chooses a control flow path based upon whether the current action belongs to an arbitrary subset A of the current possible actions. If we want to store or manipulate the current action, we use the primitive $x \rightarrow E$ to bind the current action to the variable x, which may be used in E (intuitively, this takes the place of pattern matching). To invoke one of the atomic program actions, we explicitly write $\texttt{ins}(a)$. Finally, for each of the policy combinators discussed in the previous section, we add a corresponding computation. Each of these computations is superscripted with the regulated sets for their subcomputations. Figure 2 presents a formal syntax for our language. Note that the two unnamed constructs that describe sequential conjunction ($E_1 \bigtriangleup^{A_1,A_2} (\beta \blacktriangleright E_2)$) and disjunction ($E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2)$) are used only at runtime. They encode the behavior of the first computation; in particular, whether it has inserted or accepted the action currently being considered by the second computation. The β can be either $\texttt{ins}(a)$ or $\texttt{acc}(a)$. The run-time let form ($\texttt{let}\ \{x\} = (a \triangleright E_1)^A\ \texttt{in}\ E_2$) is used when evaluating the suspended computation E_1 on an action a that is in E_1's regulated set A.

3.2 Operational Semantics

Terms We define execution of pure terms using a small-step semantics defined in terms of contexts (see Appendix A.1). The notation $C[M]$ indicates that the hole in the context C has been filled with term M. Most of the rules

(Types)	τ	$::=$	$\mathsf{act}(A) \mid \tau_1 \to \tau_2 \mid \mathsf{unit}$
		$\mid$	$\tau_1 \times \tau_2 \mid \tau_1 + \tau_2 \mid \mathcal{M}_{A_2}^{A_1}(\tau)$
(Behaviors)	β	$::=$	$\cdot \mid \mathtt{ins}(a) \mid \mathtt{sup}(a) \mid \mathtt{acc}(a)$
(Terms)	M	$::=$	x (variable)
		$\mid$	a (action)
		$\mid$	$\mathtt{fun}\ f{:}\tau\ (x).M$ (recursive function)
		$\mid$	$M_1\ M_2$ (application)
		$\mid$	$()$ (unit)
		$\mid$	$\langle M_1, M_2 \rangle$ (pairing)
		$\mid$	$\pi_1\ M \mid \pi_2\ M$ (first/second projections)
		$\mid$	$\mathtt{inl}_\tau(M_1) \mid \mathtt{inr}_\tau(M_2)$ (left/right injections)
		$\mid$	$\mathtt{case}\ M_1\ (x \to M_2 \mid x \to M_3)$ (case)
		$\mid$	$\{E\}^A$ (simple policy)
		$\mid$	$\top$ (trivially satisfiable policy)
		$\mid$	$M_1 \wedge M_2$ (parallel-conjunctive policy)
		$\mid$	$M_1 \triangle M_2$ (sequential-conjunctive policy)
		$\mid$	$\bot$ (unsatisfiable policy)
		$\mid$	$M_1 \vee_\tau M_2$ (parallel-disjunctive policy)
		$\mid$	$M_1 \triangledown_\tau M_2$ (sequential-disjunctive policy)
(Values)	v	$::=$	$x \mid a \mid \mathtt{fun}\ f{:}\tau\ (x).M \mid ()$
		$\mid$	$\langle v_1, v_2 \rangle \mid \mathtt{inl}_\tau(v_1)$
		$\mid$	$\mathtt{inr}_\tau(v_2) \mid \{E\}^A$
(Computations)	E	$::=$	M (return)
		$\mid$	$\mathtt{let}\ \{x\} = M\ \mathtt{in}\ E$ (let)
		$\mid$	$\mathtt{let}\ \{x\} = (a \triangleright E_1)^A\ \mathtt{in}\ E_2$ (run-time let)
		$\mid$	$\mathtt{ok}; E$ (accept action)
		$\mid$	$\mathtt{sup}; E$ (suppress action)
		$\mid$	$\mathtt{ins}(M); E$ (call action)
		$\mid$	$(\mathtt{next} \to E_1 \mid \mathtt{done} \to E_2)$ (next action)
		$\mid$	$x\ \to E$ (bind action)
		$\mid$	$\mathtt{acase}\ (\star\ \subseteq A)\ (E_1 \mid E_2)$ (action case)
		$\mid$	$\mathtt{case}\ M\ (x \to E_1 \mid x \to E_2)$ (case)
		$\mid$	$\mathtt{any}$ (trivial computation)
		$\mid$	$E_1 \wedge^{A_1,A_2} E_2$ (parallel-conjunctive computation)
		$\mid$	$E_1 \triangle^{A_1,A_2} E_2$ (sequential-conjunctive computation)
		$\mid$	$E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright E_2)$
		$\mid$	$\mathtt{halt}$ (terminal computation)
		$\mid$	$E_1 \vee_\tau^{A_1,A_2} E_2$ (parallel-disjunctive computation)
		$\mid$	$E_1 \triangledown_\tau^{A_1,A_2} E_2$ (sequential-disjunctive computation)
		$\mid$	$E_1 \triangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2)$

Figure 2: Syntax

are standard evaluation rules for the simply typed lambda calculus. The four rules that describe evaluation of our combinators and the two rules for the trivially satisfiable and the unsatisfiable policies (Figure 3) reduce terms to the corresponding suspended computations.

$$\top \longmapsto_\beta \{\mathtt{any}\}^\emptyset \qquad \text{(M-Top)}$$

$$\{E_1\}^{A_1} \wedge \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \wedge^{A_1,A_2} E_2\}^{A_1 \cup A_2} \qquad \text{(M-ParCon)}$$

$$\{E_1\}^{A_1} \triangle \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \triangle^{A_1,A_2} E_2\}^{A_1 \cup A_2} \qquad \text{(M-SeqCon)}$$

$$\bot \longmapsto_\beta \{\mathtt{halt}\}^\emptyset \qquad \text{(M-Bot)}$$

$$\{E_1\}^{A_1} \vee_\tau \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \vee_\tau^{A_1,A_2} E_2\}^{A_1 \cup A_2} \qquad \text{(M-ParDis)}$$

$$\{E_1\}^{A_1} \bigtriangledown_\tau \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \bigtriangledown_\tau^{A_1,A_2} E_2\}^{A_1 \cup A_2} \qquad \text{(M-SeqDis)}$$

Figure 3: Dynamic Semantics: Terms (selected rules)

Computations The evaluation rules for computations are quite lengthy, so we only present a few of them here. The remaining rules appear in Appendix A.2. This small-step semantics depends upon system states, which are pairs of a sequence of target program actions σ and a monitoring computation E which we write $\sigma \triangleright E$. We write the empty sequence as $\cdot$ and concatenate two sequences σ_1 and σ_2 using the notation $\sigma_1; \sigma_2$. Finally, we write $\sigma \notin A$ to indicate that no actions in the sequence σ appear in the set of actions A.

The rules have the form $\vdash^{A_r} \sigma_1 \triangleright E_1 \overset{\beta}{\longmapsto} \sigma_2 \triangleright E_2$, where A_r is the regulated set for this computation, $\sigma_1 \triangleright E_1$ is the initial system state, $\sigma_2 \triangleright E_2$ is the final state, and β is the effect of the transition (i.e., insertion of an action, suppression of an action, acceptance of an action, or nothing). When the effect β is $\mathtt{ins}(a)$ or $\mathtt{acc}(a)$, the action a is emitted by the monitor at this step of the execution.

The dynamic semantics for the some of the basic computational operators are given below.

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{ok}; E \overset{\mathtt{acc}(a)}{\longmapsto} \sigma \triangleright E} \qquad \text{(E-Acc)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{sup}; E \overset{\mathtt{sup}(a)}{\longmapsto} \sigma \triangleright E} \qquad \text{(E-Sup)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{ins}(a); E \overset{\mathtt{ins}(a)}{\longmapsto} \sigma \triangleright E} \qquad \text{(E-Ins2)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright (\texttt{next} \to E_1 \mid \texttt{done} \to E_2) \stackrel{\cdot}{\longmapsto} a; \sigma \triangleright E_1} \quad \text{(E-Next)}$$

$$\frac{a \notin A_r}{\vdash^{A_r} a; \sigma \triangleright (\texttt{next} \to E_1 \mid \texttt{done} \to E_2) \stackrel{\mathsf{acc}(a)}{\longmapsto} \sigma \triangleright (\texttt{next} \to E_1 \mid \texttt{done} \to E_2)} \quad \text{(E-Next-Skip)}$$

$$\frac{}{\vdash^{A_r} \cdot \triangleright (\texttt{next} \to E_1 \mid \texttt{done} \to E_2) \stackrel{\cdot}{\longmapsto} \cdot \triangleright E_2} \quad \text{(E-Done)}$$

The rules (E-Acc) and (E-Sup) are fairly straightforward. One key difference between the two is that the action a appears in the output in the former case but not in the latter case (it is suppressed). Both rules require the input stream to be non-empty and the first action in the stream to be part of the regulated set. We consider it *wrong* for a monitor to act on or suppress actions that are not in its regulated set. The static semantics will ensure that such errors do not occur. We do, however, allow the monitor to call actions not in its regulated set (rule (E-Ins2)).

The `next` construct operates similarly to a `case` statement. Rule (E-Next) indicates that we take the first branch when the next action belongs to the regulated set and (E-Done) indicates we take the second branch when there are no more actions. (E-Next-Skip) specifies that we skip actions that do not appear in the regulated set.

The rule for the trivial computation and the rules for parallel and sequential conjunction are given below.

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathsf{any} \stackrel{\cdot}{\longmapsto} \sigma \triangleright ()} \quad \text{(E-Any)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \|\beta\| \notin A_2 \text{ or } (E_2 = v \text{ and } \beta = \mathsf{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1, A_2} E_2) \stackrel{\beta}{\longmapsto} \sigma' \triangleright (E_1' \wedge^{A_1, A_2} E_2)} \quad \text{(E-ParCon1)}$$

$$\frac{\vdash^{A_2} \sigma \triangleright E_2 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_2' \quad \|\beta\| \notin A_1 \text{ or } (E_1 = v \text{ and } \beta = \mathsf{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1, A_2} E_2) \stackrel{\beta}{\longmapsto} \sigma' \triangleright (E_1 \wedge^{A_1, A_2} E_2')} \quad \text{(E-ParCon2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\mathsf{acc}(a)}{\longmapsto} \sigma' \triangleright E_1' \quad \vdash^{A_2} \sigma \triangleright E_2 \stackrel{\mathsf{acc}(a)}{\longmapsto} \sigma' \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1, A_2} E_2) \stackrel{\mathsf{acc}(a)}{\longmapsto} \sigma' \triangleright (E_1' \wedge^{A_1, A_2} E_2')} \quad \text{(E-ParCon3)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v_1 \wedge^{A_1, A_2} v_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \langle v_1, v_2 \rangle} \quad \text{(E-ParCon4)}$$

$$\frac{E_1 = \texttt{halt} \text{ or } E_2 = \texttt{halt}}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1, A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \texttt{halt}} \quad \text{(E-ParCon5)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{sup}(a) \text{ or } \beta = \cdot}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1' \mathbin{\triangle^{A_1,A_2}} E_2)} \quad \text{(E-SeqCon1)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \notin A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1' \mathbin{\triangle^{A_1,A_2}} E_2)} \quad \text{(E-SeqCon2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \in A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\cdot}{\longmapsto} \sigma' \triangleright (E_1' \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright E_2))} \quad \text{(E-SeqCon3)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \overset{\beta'}{\longmapsto} \|\beta\| \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright E_2)) \overset{\beta'}{\longmapsto} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright E_2'))} \quad \text{(E-SeqCon4)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \overset{\beta'}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright E_2)) \overset{\beta''}{\longmapsto} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} E_2')} \quad \text{(E-SeqCon5)}$$

$$\text{where } \beta'' = \begin{cases} \cdot & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{ins}(a) & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{acc}(a) \\ \mathtt{sup}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{acc}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{acc}(a) \end{cases}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright v)) \overset{\beta}{\longmapsto} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} v)} \quad \text{(E-SeqCon6)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright \mathtt{halt})) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-SeqCon7)}$$

$$\frac{E_2 \neq v \quad E_2 \neq \mathtt{halt}}{\vdash^{A_r} a; \sigma \triangleright (v \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\cdot}{\longmapsto} \sigma \triangleright (v \mathbin{\triangle^{A_1,A_2}} (\mathtt{acc}(a) \blacktriangleright E_2))} \quad \text{(E-SeqCon8)}$$

$$\frac{\vdash^{A_2} \cdot \triangleright E_2 \overset{\beta}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \cdot \triangleright (v \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\beta}{\longmapsto} \cdot \triangleright (v \mathbin{\triangle^{A_1,A_2}} E_2')} \quad \text{(E-SeqCon9)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v_1 \mathbin{\triangle^{A_1,A_2}} v_2) \overset{\cdot}{\longmapsto} \sigma \triangleright \langle v_1, v_2 \rangle} \quad \text{(E-SeqCon10)}$$

$$\frac{E_1 = \mathtt{halt} \text{ or } E_2 = \mathtt{halt}}{\vdash^{A_r} \sigma \triangleright (E_1 \mathbin{\triangle^{A_1,A_2}} E_2) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-SeqCon11)}$$

The trivial computation immediately returns () without effect (E-Any). The first rule for parallel conjunction (E-ParCon1) allows the first subcomputation (E_1) to call functions and suppress actions that do not appear in the regulated set of the second subcomputation (E_2). We use the notation $\|\beta\|$ to extract

the action inserted, accepted, or suppressed by E_1. Formally, we define $\|\beta\|$ as follows.

$$\|\beta\| = \begin{cases} a & \text{if } \beta = \texttt{ins}(a) \\ a & \text{if } \beta = \texttt{acc}(a) \\ a & \text{if } \beta = \texttt{sup}(a) \\ \cdot & \text{if } \beta = \cdot \end{cases}$$

This rule (E-ParCon1) states one of the main safety conditions for our system: E_1 may not interfere with E_2 by inserting or suppressing actions that should be regulated by E_2. If the first subcomputation wants to accept an action that is in the second subcomputation's regulated set, and the second subcomputation has already evaluated to a value, the conjunction automatically accepts the action. A symmetric rule (E-ParCon2) allows the second subcomputation to execute in an analogous way. When an action is in the regulated set of both subcomputations, and one accepts it, so too must the other (E-ParCon3). The last two rules rules for parallel conjunction ((E-ParCon4) and (E-ParCon5)) state the termination conditions for a conjunctive policy. When both subcomputations produce a value, the conjunction will produce a pair. When either subcomputation decides that the target should be terminated, the conjunction will halt the target.

In sequential conjunction, the target input is fed into the first subcomputation, E_1. E_1 feeds its output, an action a that it has either inserted or accepted, into E_2 if E_2 regulates a. Execution of the sequential conjunction alternates between execution of the first subcomputation and, when it produces some output, execution of the second subcomputation until it has consumed that output. To describe this behavior, we introduce the runtime form $E_1 \mathbin{\triangle^{A_1,A_2}} (\beta \blacktriangleright E_2)$, which we use to keep track of whether the action a being considered by E_2 is the result of insertion ($\texttt{ins}(a)$) or acceptance ($\texttt{acc}(a)$) by E_1.

The first rule for sequential conjunction (E-SeqCon1) states that as long as the first subcomputation neither accepts nor inserts an action, the second subcomputation has no input to process, so the behavior of the composition is determined solely by the behavior of the first subcomputation. If the output of the first subcomputation is not regulated by the second subcomputation, again the behavior of the first subcomputation determines the behavior of the composition (E-SeqCon2). On the other hand, if the second subcomputation regulates the output of the first subcomputation, this output is fed to the second subcomputation (E-SeqCon3), whose behavior determines the behavior of the composition (E-SeqCon4 and E-SeqCon5). The second subcomputation can consume the action fed to it by the first subcomputation by either accepting or suppressing it (E-SeqCon5). If E_2 suppresses an action a inserted by E_1, the composition emits nothing; if a was accepted by E_1, the composition suppresses a. If E_2 accepts an action a inserted by E_1, the composition inserts a; if a was accepted by E_1, the composition accepts a. The remaining rules for sequential conjunction describe cases when one of the subcomputations has evaluated to a value or halted.

The disjunctive combinators (see Appendix A.2) are very similar to their

conjunctive counterparts, except for their termination conditions. When either subcomputation produces a value, the disjunction will inject the value into a sum and return. On the other hand, both subcomputations must decide to terminate the target in order for the disjunction to halt the target.

The operational semantics for the let construct (see Appendix A.2) specifies that the suspended computation is first evaluated with respect to the input stream. The value it returns is bound to a variable, and execution continues with the body of the let computation.

System Execution A running system $\mathcal{S}$ is a sequence of program actions paired with a monitoring computation and some set of regulated actions. A fully evaluated system is either a value (indicating that the monitor processed all input actions and the computation is fully evaluated) or the special `halt` symbol (indicating that the monitor terminated the target while processing the input sequence).

$$\mathcal{S} ::= \mathsf{run}(\sigma, A_r, E) \mid v \mid \mathtt{halt}$$

We represent system execution using the judgment $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$ where σ represents the final observable output of the system.

$$\frac{a \notin A_r}{\mathsf{run}(a; \sigma, A_r, E) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma, A_r, E)} \quad \text{(R-Skip)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \quad \sigma = \cdot \text{ or } (\sigma = a'; \sigma'' \text{ and } a' \in A_r) \quad \beta = \cdot \text{ or } \beta = \mathtt{sup}(a)}{\mathsf{run}(\sigma, A_r, E) \dot{\hookrightarrow} \mathsf{run}(\sigma', A_r, E')} \quad \text{(R-Step1)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \quad \sigma = \cdot \text{ or } (\sigma = a'; \sigma'' \text{ and } a' \in A_r) \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a)}{\mathsf{run}(\sigma, A_r, E) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma', A_r, E')} \quad \text{(R-Step2)}$$

$$\frac{}{\mathsf{run}(a; \sigma, A_r, v) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma, A_r, v)} \quad \text{(R-Val1)}$$

$$\frac{}{\mathsf{run}(\cdot, A_r, v) \dot{\hookrightarrow} v} \quad \text{(R-Val2)}$$

$$\frac{}{\mathsf{run}(\sigma, A_r, \mathtt{halt}) \dot{\hookrightarrow} \mathtt{halt}} \quad \text{(R-Halt)}$$

3.3 Static Semantics

We specify the static semantics for the language using four main judgments.

Subtyping: $\vdash \tau_1 \leq \tau_2$ The rules for subtyping are mostly standard (see Appendix B.1). Unit, pairs, sums and function types have their usual subtyping rules. We say the type of actions $\mathsf{act}(A)$ is covariant in A since $\mathsf{act}(A)$ is a subtype of $\mathsf{act}(A')$ when $A \subseteq A'$. Policy types are covariant in their return type and effect set but invariant in their regulated set. In other words, it is safe for policies to appear to have a larger effect than they actually do, but they must regulate the set that they claim to regulate.

Term Typing: $\Gamma \vdash M : \tau$ The term typing rules contain the ordinary introduction and elimination rules for functions, unit, pairs and sums (see Appendix B.2). The treatment of variables is also standard. The basic rule for actions gives an action a the singleton type $\mathsf{act}(\{a\})$. When this rule is used in conjunction with the subsumption rule, an action may be given any type $\mathsf{act}(A)$ such that $a \in A$. The typing rules for policy terms are given below.

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A_e}{\Gamma \vdash \{E\}^{A_r} : \mathcal{M}^{A_r}_{A_e}(\tau)} \quad \text{(S-Sus)}$$

$$\frac{}{\Gamma \vdash \top : \mathcal{M}^{\emptyset}_{\emptyset}(\mathsf{unit})} \quad \text{(S-Top)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2) \quad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \wedge M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2)} \quad \text{(S-ParCon)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)}{\Gamma \vdash M_1 \triangle M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2)} \quad \text{(S-SeqCon)}$$

$$\frac{}{\Gamma \vdash \bot : \mathcal{M}^{\emptyset}_{\emptyset}(\tau)} \quad \text{(S-Bot)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2) \quad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \vee_{\tau_1+\tau_2} M_2 : \mathcal{M}^{A_1 \cup A_2}_{A_3 \cup A_4}(\tau_1 + \tau_2)} \quad \text{(S-ParDis)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)}{\Gamma \vdash M_1 \bigtriangledown_{\tau_1+\tau_2} M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 + \tau_2)} \quad \text{(S-SeqDis)}$$

Elementary policies (rule (S-Sus)) are given the type $\mathcal{M}^{A_r}_{A_e}(\tau)$ when the suspended computation regulates the actions in A_r, has effect A_e and produces a value of type τ. The trivial policy (rule (S-Top)) makes its decisions based upon no regulated actions, has no effect and simply returns unit. The terminal policy (rule (S-Bot)) also makes its decision based upon no regulated actions, has no effect, but instead of returning a value, it immediately calls for termination of the target. Since the terminal policy never returns, we allow its return type to be any type τ.

Rules (S-ParCon) and (S-SeqCon) give types to the two conjunctive policies. In each case, the type of the resulting computation involves taking the union of the regulated sets and the union of the effects since a conjunctive policy makes its decisions based on the regulated actions of both policies and potentially has the effects of either policy. These combinators return a pair of values, which is reflected in the type of the conjunctive combinator. The parallel conjunction is constrained so that the regulated set of one conjunct is disjoint from the effect of the other and vice versa. This constraint prevents one conjunct from inserting or suppressing actions that should be regulated by the other conjunct. Typing for the sequential conjunction is more liberal. It allows one policy to supersede another regardless of the effects of either policy. The rules for the disjunctive combinators ((S-ParDis) and (S-SeqDis)) are analogous to their conjunctive counterparts except that disjunctions return sums rather than pairs.

Computation Typing: $\Gamma; B \vdash^{A_r} E : \tau, A_e$ The basic judgment for typing computations may be read "Computation E produces a value with type τ and has effect A_e in Γ when run against a target whose next action is in B." B ranges over non-empty sets A or the symbol $\diamond$, which represents no knowledge about the next action. The next action might not even exist, as is the case when the target has terminated. We maintain this set of possible next actions so that we know what actions to consider as possible effects of a suppress statement and what actions may be bound to a variable in a `bind` statement. We do not consider computation judgments to be valid unless either $B \subseteq A_r$ or $B = \diamond$. We define $B \sqcup A_r$ to be B if $B \subseteq A_r$ and $\diamond$ otherwise. Finally, set intersect and set minus operators $\cap_\diamond$ and $\setminus_\diamond$ act like standard set operators, except that instead of returning $\emptyset$ they return $\diamond$.

The computation typing rules are given below.

$$\frac{\Gamma \vdash M : \tau}{\Gamma; B \vdash^{A_r} M : \tau, \emptyset} \quad \text{(SE-Ret)}$$

$$\frac{\Gamma \vdash M : \mathcal{M}^{A'_r}_{A_2}(\tau') \quad \Gamma, x{:}\tau'; \diamond \vdash^{A_r} E : \tau, A \quad A'_r \subseteq A_r}{\Gamma; B \vdash^{A_r} \texttt{let}\ \{x\} = M \texttt{in}\ E : \tau, A \cup A_2} \quad \text{(SE-Let1)}$$

$$\frac{\cdot; \{a\} \vdash^{A} E_1 : \tau', A_1 \quad x{:}\tau'; \diamond \vdash^{A_r} E_2 : \tau, A_2 \quad A \subseteq A_r \quad a \in A}{\cdot; \{a\} \vdash^{A_r} \texttt{let}\ \{x\} = (a \triangleright E_1)^{A} \texttt{in}\ E_2 : \tau, A_1 \cup A_2} \quad \text{(SE-Let2)}$$

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} \texttt{ok}; E : \tau, A} \quad \text{(SE-Acc)}$$

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} \texttt{sup}; E : \tau, A \cup B} \quad \text{(SE-Sup)}$$

$$\frac{\Gamma \vdash M : \mathsf{act}(A') \quad \Gamma; B \vdash^{A_r} E : \tau, A}{\Gamma; B \vdash^{A_r} \texttt{ins}(M); E : \tau, A \cup A'} \quad \text{(SE-Ins)}$$

$$\frac{\Gamma; A_r \vdash^{A_r} E_1 : \tau, A \quad \Gamma; \diamond \vdash^{A_r} E_2 : \tau, A}{\Gamma; B \vdash^{A_r} (\texttt{next} \to E_1 \mid \texttt{done} \to E_2) : \tau, A} \quad \text{(SE-Next)}$$

$$\frac{\Gamma, x{:}\mathsf{act}(B); B \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} x \ \to E : \tau, A} \quad \text{(SE-Bind)}$$

$$\frac{\begin{array}{c}\Gamma; B \cap_\diamond A' \vdash^{A_r} E_1 : \tau, A \qquad \Gamma; B \setminus_\diamond A' \vdash^{A_r} E_2 : \tau, A \\ A' \subseteq A_r \qquad B \neq \diamond\end{array}}{\Gamma; B \vdash^{A_r} \texttt{acase}\ (\star \ \subseteq A')\ (E_1 \mid E_2) : \tau, A} \quad \text{(SE-Acase)}$$

$$\frac{\begin{array}{c}\Gamma \vdash M : \tau_1 + \tau_2 \\ \Gamma, x{:}\tau_1; B \vdash^{A_r} E_1 : \tau, A \qquad \Gamma, x{:}\tau_2; B \vdash^{A_r} E_2 : \tau, A\end{array}}{\Gamma; B \vdash^{A_r} \texttt{case}\ M\ (x \to E_1 \mid x \to E_2) : \tau, A} \quad \text{(SE-Case)}$$

$$\frac{}{\Gamma; B \vdash^{A_r} \texttt{any} : \mathsf{unit}, \emptyset} \quad \text{(SE-Any)}$$

$$\frac{\begin{array}{c}\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \Gamma; B \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cap A_4 = A_2 \cap A_3 = \emptyset \qquad A_1 \cup A_2 = A_r\end{array}}{\Gamma; B \vdash^{A_r} E_1 \wedge^{A_1, A_2} E_2 : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-ParCon)}$$

$$\frac{\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \quad \Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4 \quad A_1 \cup A_2 = A_r}{\Gamma; B \vdash^{A_r} E_1 \triangle^{A_1, A_2} E_2 : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon1)}$$

$$\frac{\begin{array}{c}\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cup A_2 = A_r \qquad a \in A_2 \qquad a \in A_3\end{array}}{\cdot; B \vdash^{A_r} E_1 \triangle^{A_1, A_2} (\texttt{ins}(a) \blacktriangleright E_2) : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon2)}$$

$$\frac{\begin{array}{c}\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cup A_2 = A_r \qquad a \in A_2\end{array}}{\cdot; B \vdash^{A_r} E_1 \triangle^{A_1, A_2} (\texttt{acc}(a) \blacktriangleright E_2) : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon3)}$$

$$\frac{}{\Gamma; B \vdash^{A_r} \texttt{halt} : \tau, \emptyset} \quad \text{(SE-Halt)}$$

$$\frac{\begin{array}{c}\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \Gamma; B \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cap A_4 = A_2 \cap A_3 = \emptyset \qquad A_1 \cup A_2 = A_r\end{array}}{\Gamma; B \vdash^{A_r} E_1 \vee^{A_1, A_2}_{\tau_1 + \tau_2} E_2 : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-ParDis)}$$

$$\frac{\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \quad \Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4 \quad A_1 \cup A_2 = A_r}{\Gamma; B \vdash^{A_r} E_1 \bigtriangledown^{A_1, A_2}_{\tau_1 + \tau_2} E_2 : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis1)}$$

$$\frac{\begin{array}{c}\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cup A_2 = A_r \qquad a \in A_2 \qquad a \in A_3\end{array}}{\cdot; B \vdash^{A_r} E_1 \bigtriangledown^{A_1, A_2}_{\tau_1 + \tau_2} (\texttt{ins}(a) \blacktriangleright E_2) : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis2)}$$

$$\frac{\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \quad A_1 \cup A_2 = A_r \quad a \in A_2}{\cdot; B \vdash^{A_r} E_1 \bigtriangledown^{A_1,A_2}_{\tau_1+\tau_2} (\mathtt{acc}(a) \blacktriangleright E_2) : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis3)}$$

$$\frac{\Gamma; B' \vdash^{A_r} E : \tau', A' \quad (B' = \diamond \ \ \mathtt{or} \ \ B \subseteq B') \quad \vdash \tau' \leq \tau \quad A' \subseteq A}{\Gamma; B \vdash^{A_r} E : \tau, A} \quad \text{(SE-Sub)}$$

Terms have no effects, so they are well typed with respect to any next action (SE-Ret).

The `let` rule (SE-Let1) requires M to be a policy with a regulated set that is a subset of the current computation's regulated set. When this policy returns, we will have no information regarding the next action because the suspended policy may have accepted or suppressed an arbitrary number of actions. As a result, we check E in a context involving $\diamond$. The secondary let rule (SE-Let2) records a few further invariants necessary for the proof of safety, namely, that the suspended computation E_1 is well typed with respect to the action it is currently processing, and that that action is an element of E_1's regulated set.

Rules (SE-Acc) and (SE-Sup) have similar structure. In both cases, we must be sure that the target has produced some action to be accepted or suppressed (i.e., $B \neq \diamond$). The main difference between the two rules is that we record the effect of the suppression, whereas acceptance has no effect. The rule for invoking actions (SE-Ins) adds A' to the effect of the computation when the action called belongs to the set A' (in other words, when the action has type $\mathtt{act}(A')$).

The next/done construct adds A_r to the context for checking E_1 and $\diamond$ for checking E_2 since we only take the first branch when we see an action in the regulated set and we take the second branch when there are no more actions (rule (SE-Next)). Rule (SE-Acase) takes the first or second branch depending upon whether the current action is in the set A_1. We refine the context in each branch to reflect the information that we have about the current action.

Rule (SE-ParCon) places several constraints on parallel conjunction of computations. Since the next action could be in the regulated set of the conjunction but not in the regulated sets of both E_1 and E_2, E_1 and E_2 must both be well typed either with respect to a subset of their regulated sets or with respect to $\diamond$. This is ensured by typing the subcomputations with respect to $B \sqcup A_1$ and $B \sqcup A_2$. In addition, there is not allowed to be a conflict between the regulated actions of one subcomputation and the effects of the other. Finally, the regulated set of the conjunction must be the union of the regulated sets of the subcomputations.

The first rule for sequential conjunction (SE-SeqCon1) is similar, with two exceptions. First, there is no constraint on the regulated and effect sets of the subcomputations. Second, E_2 must be well typed with respect to $\diamond$ because we cannot make any assumption about what the next action will be (it may be an action emitted by E_1, or E_1 may suppress all actions until the target has finished executing). The second sequential conjunction rule (SE-SeqCon2)

records several run-time invariants. The second subcomputation is well typed with respect to any action a fed to it by the first subcomputation. In addition, a must be in E_2's regulated set and, because a was inserted by E_1, in the effect set of the well-typed first subcomputation. Rule (SE-SeqCon3) is similar and describes the situation when a was accepted by E_1.

The rules for the disjunctive operators (SE-ParDis and SE-SeqDis1 through SE-SeqDis3) are identical to their conjunctive counterparts except that they have sum types rather than pair types.

The subsumption rule for computations (SE-Sub) is invariant in regulated sets, covariant in type and effect sets, and contravariant in the type of the next action. It is always OK to consider that a computation has more effects than it actually does. In addition, a computation typed with respect to the possible next actions B' continues to be well typed even if more information about the next action is available.

System Typing The rules for concluding that a running system is well formed depend upon an auxiliary judgment for giving types to the first action in a sequence ($\vdash \sigma : A$).

$$\frac{\vdash \sigma : B \quad \cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A}{\vdash^{A_r} \sigma \triangleright E \ \mathtt{ok}} \qquad \text{(S-Ok)}$$

$$\frac{}{\vdash \cdot : \diamond} \qquad \text{(Seq-Empty)}$$

$$\frac{}{\vdash a; \sigma : \{a\}} \qquad \text{(Seq-Act)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \ \mathtt{ok}}{\vdash \mathsf{run}(\sigma, A_r, E) \ \mathtt{ok}} \qquad \text{(Run-Ok)}$$

$$\frac{\vdash v : \tau}{\vdash v \ \mathtt{ok}} \qquad \text{(Val-Ok)}$$

$$\frac{}{\vdash \mathtt{halt\ ok}} \qquad \text{(Halt-Ok)}$$

3.4 Properties

Because this calculus describes high-level running systems as somewhat complex compositions of action streams, sets of regulated actions, and computations (which may themselves be terms), the proof of type safety is necessarily lengthy and rather intricate. The safety proof for running systems is built upon proofs of safety for terms and computations, with each of these proofs requiring canonical forms, inversion of typing, preservation, and progress lemmas. All the lemmas are given in Appendix C. We also provide proof outlines and, for non-trivial cases, actual proofs.

Lemma 24 (Preservation: Running System)
If $\vdash \mathcal{S}$ `ok` *and* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$ *then* $\vdash \mathcal{S}'$ `ok`.

Lemma 25 (Progress: Running System)
If $\vdash \mathcal{S}$ `ok` *then* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$, *or* $\mathcal{S} = v$, *or* $\mathcal{S} =$ `halt`.

Definition 26 ($\overset{\sigma}{\hookrightarrow}^*$)
We extend the single-step dynamic semantics of running systems to a multi-step semantics with the usual reflexive and transitive rules.

$$\frac{}{\mathcal{S} \overset{\cdot}{\hookrightarrow}^* \mathcal{S}} \qquad \text{(R*-Reflex)}$$

$$\frac{\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}' \quad \mathcal{S}' \overset{\sigma'}{\hookrightarrow}^* \mathcal{S}''}{\mathcal{S} \overset{\sigma;\sigma'}{\hookrightarrow}^* \mathcal{S}''} \qquad \text{(R*-Trans)}$$

Definition 27 (Stuck System)
A system $\mathcal{S}$ *is "stuck" if* $\mathcal{S} \neq v$ *and* $\mathcal{S} \neq$ `halt` *and there does not exist a system* $\mathcal{S}'$ *such that* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$.

Theorem 28 (System Safety)
If $\vdash \mathcal{S}$ `ok` *and* $\mathcal{S} \overset{\sigma}{\hookrightarrow}^* \mathcal{S}'$ *then* $\vdash \mathcal{S}'$ `ok` *and* $\mathcal{S}'$ *is not stuck.*

4 Implementation

In order to confirm that our policy calculus is feasible and useful, we have developed a practical implementation of it. Polymer, our language for writing policies, implements most of the policy calculus and allows the use of many of the features and most of the syntax of Java. For simplicity, the target programs we currently consider are Java source programs, but many of the techniques we use can also be extended to handle Java bytecode. We have not yet fully implemented static checking of effects.

Polymer We illustrate the main differences between our calculus and Polymer using as an example a policy that restricts the number and type of files that may be opened via a `FileWriter`.

```
polinterface SystemInterface =
   ... java.io.FileWriter(String path);
       java.io.FileWriter.close();
       java.lang.System.err.println(String s); ...
```

```
policy limitWriter(int maxopen) : SystemInterface =
  actions = { java.io.FileWriter(String path);
              java.io.FileWriter.close(); }
  state =   { int cur = 0; }
  policy =  {
              aswitch {
                case java.io.FileWriter(String path) :
                  if (cur >= maxopen) suppress;
                  if (path.startsWith("/tmp")) {
                    cur++; ok;
                  }
                  else {
                    emit(System.err.println("Can't open."));
                    suppress;
                  }
                case java.io.FileWriter.close() :
                  cur--;
                case done :
              }
            }
```

In the implemented version of the language, policies are accompanied by an interface (`polinterface SystemInterface`). Informally, an interface is the set of all security-relevant actions that we wish to regulate. Keeping the interface separate from the policies allows us to instrument a target program independently and then run it in combination with any policy whose regulated set is a subset of the interface. It also helps us to keep track of which of the Java statements executed by the policy represent effects and which are merely part of the policy's computation. This makes it relatively straightforward to statically check for interference between policies.

The first line of the policy,

```
policy limitWriter(int maxopen) : SystemInterface
```

specifies its name and parameters (in this case, an `int` representing the maximum number of files that may be opened), and that its regulated and effect sets are subsets of the set of actions listed in `SystemInterface`. The parameters can be policies or have standard Java types.

The body of the policy declaration consists of three blocks. The `actions` and `policy` blocks are as described in Section 2. The `state` block is used for declaring persistent variables and for importing other policies. Importing a policy allows it to be used within the `policy` block.

The body of the `policy` block is composed of a sequence of Java statements written as if they were in the body of an ordinary Java method. In addition to standard Java, Polymer supports several additional statements. The `aswitch` statement has syntax similar to the Java `switch` statement and allows us to pattern match against method calls. Variables from the pattern (such as the

string `path`) have as their scope the matching case block. `ok` and `suppress` have the obvious meanings. If the policy wishes to add actions to the program stream (in this case by printing an error message), it does so via the `emit` statement. Any emitted action must be in the interface that accompanies the policy; a policy can invoke an action that is in its interface only by enclosing it in an `emit` statement. `halt` signals that the target program should be terminated. In contrast to the calculus, where the recursion in a policy is explicit, in practice all our policies are implicitly recursive; that is, they will continue processing instructions until explicitly told otherwise. To stop executing a policy and return a value we use the `stop` statement. The `run` statement executes a policy that was either passed to the current policy as an argument or imported in the `state` block.

To write compound policies, one can replace the `actions`, `state` and `policy` blocks with the statement of the compound policy, e.g., a parallel conjunction (`PolicyA parallelAnd PolicyB`).

Instrumenting target programs We allow a monitor to interpose itself between the action stream and the runtime system by rewriting the method calls of a target program. A call to a monitored method is rewritten as a call to the monitor. A new method is added to the target program, which allows the monitor to execute the original call via a call-back. If it wishes to allow the method call, the monitor explicitly makes the call and then returns; to suppress it, it ignores the call and it throws a `SuppressionException`. The target program can catch the exception and amend its behavior accordingly. If it does not have an appropriate exception handler, one is added during rewriting. This allows the target program to get feedback while ensuring that the monitor is not circumvented.

Representing policies Policies are implemented as classes. We have developed a hierarchy of policy classes that contains implementations of the semantics of the various types of policies we have in our calculus. A policy written in Polymer compiles into a class that extends the appropriate policy class. For example, the class in the hierarchy that implements the semantics for sequential conjunction will be the superclass of all sequential-conjunctive policies written in Polymer.

Enforcing policies In our system the monitor is a program independent from a particular policy. To enforce a policy on a target program, we first instrument the target and compile the policy, and then run the monitor program with the target and the policy as parameters. The instrumented target passes control to the monitor at appropriate times, and the monitor forwards the call to the instantiated policy object and relays its decision back to the target. The policy communicates with the monitor by either returning (to indicate that the action which triggered the call to the monitor should be allowed to execute) or throwing an exception.

Compound policies, such as parameterized or parallel policies, communicate internally in the same manner. Sequentially composed policies are more complicated because they require that any actions emitted by the first policy are seen by the second policy before they are executed. Since the first policy in a sequence could be a compound policy which is assembled only at runtime (a parameterized policy, for example), we need a dynamic mechanism for regulating emitted actions. Hence, we implement the `emit` statement by throwing an exception that is caught by the enclosing policy or by the monitor, which then either executes the emitted action or allows another policy to decide whether the action should be executed.

5 Discussion

5.1 Related Work

The SDS-940 system at Berkeley [DG71] was the first to use code rewriting to enforce security properties. More recently, the advent of safe languages such as Java, Haskell, ML, Modula, and Scheme, which allow untrusted applications to interoperate in the same address space with system services, has led to renewed efforts to design flexible and secure monitoring systems. For example, Evans and Twyman's Naccio system [ET99] allows security architects to declare *resources*, which are security-relevant interfaces, and to attach *properties*, which are bundles of security state and checking code, to these resources. Erlingsson and Schneider's SASI language [ES99] and later Poet and Pslang system [ES00] provide similar power. Grimm and Bershad [GB01] describe and evaluate a flexible mechanism that separates the access-control mechanism from policy in the SPIN extensible operating system. Finally, the Ariel project [PH00] allows security experts to write boolean constraints that determine whether or not a method can be invoked.

A shortcoming of all these projects is a lack of formal semantics for the proposed languages and systems. Without a formal semantics, system implementers have no tools for precise reasoning about their systems. They also do not provide a general set of primitives that programmers can use to explicitly construct complex policies from simpler ones.

A slightly different approach to program monitoring is taken by Lee et al. [KVBA+99, LKK+99] and Sandholm and Schwarzbach [SS98]. Rather than writing an explicit program to monitor applications as we do, they specify the safety property in which they are interested either in a specialized temporal logic (Lee et al.) or second-order monadic logic (Sandholm and Schwarzbach).

Many monitoring systems may be viewed as a specialized form of aspect-oriented programming. Aspect-oriented programming languages such as AspectJ [KHH+01] allow programmers to specify *pointcuts*, which are collections of program points and *advice*, which is code that is inserted at a specified pointcut. Wand et al. [WKD02] give a denotational semantics for these features using monadic operations. Conflicting advice inserted at the same pointcut is a known

problem in aspect-oriented programming. AspectJ solves the problem by specifying a list of rules that determine the order in which advice will be applied. We believe that our language, which allows explicit composition of policies and makes it possible to statically check composed policies for interference, is a more flexible approach to solving this problem.

Theoretical work by Alpern and Schneider [AS87, Sch00] gives an automaton-theoretic characterization of safety, liveness, and execution monitoring (EM) policies. EM policies are the class of policies enforceable by a general-purpose program monitor that may terminate the target, but may not otherwise modify target behavior. This class of program monitors (called security automata) corresponds precisely to our effect-free monitors, and consequently, as pointed out by Schneider, they are easily composed. We have previously extended Schneider's work by defining a new class of automata [BLW02a, BLW02b], the *edit automata*, which are able to insert and suppress target actions as well as terminate the target. Edit automata more accurately characterize practical security monitors that modify program behavior. We proved such automata are strictly more powerful than security automata.

5.2 Current and Future Work

Our immediate concern is to acquire more experience applying our tool to enforcing security policies on realistic applications. We are interested both in testing our tool on untrusted mobile programs as well as using it to make programs and services written by trusted programmers more robust. As an example of the latter application, we intend to follow Qie et al. [QPP02] and use our tool to control resource consumption and to help prevent denial of service in Web servers.

Rather than having an external tool that rewrites Java programs to enforce policies, we plan to internalize the rewriting process within an extension to the Java language. We hope to develop techniques that allow programmers to dynamically rewrite programs or parts of programs and to update or modify security policies without necessarily bringing down the system. We believe the idea of policies as first-class objects will be crucial in this enterprise.

We plan to investigate additional combinators that could be added to our language. In particular, we are interested in developing a precise semantics for fixed point combinators that extend our sequential operators. This would make it possible to iteratively combine two policies without restricting their effects or requiring that one supersedes the other.

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A Dynamic Semantics

A.1 Terms

$$\begin{array}{lrl}
\text{(Context)} & C ::= & [\,] \mid C\ M \mid v\ C \mid \langle C, M\rangle \mid \langle v, C\rangle \\
& \mid & \pi_1\ C \mid \pi_2\ C \mid \mathtt{inl}_\tau(C) \mid \mathtt{inr}_\tau(C) \\
& \mid & \mathtt{case}\ C\ (x \to M_1 \mid x \to M_2) \\
& \mid & C \wedge M_2 \mid v \wedge C \mid C \triangle M_2 \mid v \triangle C \\
& \mid & C \vee_\tau M_2 \mid v \vee_\tau C \mid C \bigtriangledown_\tau M_2 \mid v \bigtriangledown_\tau C
\end{array}$$

$$\mathtt{fun}\ f{:}\tau(x).M\ v \longmapsto_\beta [\mathtt{fun}\ f{:}\tau(x).M/f][v/x]M \qquad \text{(M-App)}$$

$$\pi_1\ \langle v_1, v_2\rangle \longmapsto_\beta v_1 \qquad \text{(M-Proj1)}$$

$$\pi_2\ \langle v_1, v_2\rangle \longmapsto_\beta v_2 \qquad \text{(M-Proj2)}$$

$$\mathtt{case}\ \mathtt{inl}_\tau(v)\ (x \to M_1 \mid x \to M_2) \longmapsto_\beta [v/x]M_1 \qquad \text{(M-CaseL)}$$

$$\mathtt{case}\ \mathtt{inr}_\tau(v)\ (x \to M_1 \mid x \to M_2) \longmapsto_\beta [v/x]M_2 \qquad \text{(M-CaseR)}$$

$$\top \longmapsto_\beta \{\mathtt{any}\}^\emptyset \qquad \text{(M-Top)}$$

$$\{E_1\}^{A_1} \wedge \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \wedge^{A_1,A_2} E_2\}^{A_1 \cup A_2} \qquad \text{(M-ParCon)}$$

$$\{E_1\}^{A_1} \triangle \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \triangle^{A_1,A_2} E_2\}^{A_1 \cup A_2} \quad \text{(M-SeqCon)}$$

$$\bot \longmapsto_\beta \{\texttt{halt}\}^{\emptyset} \quad \text{(M-Bot)}$$

$$\{E_1\}^{A_1} \vee_\tau \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \vee_\tau^{A_1,A_2} E_2\}^{A_1 \cup A_2} \quad \text{(M-ParDis)}$$

$$\{E_1\}^{A_1} \bigtriangledown_\tau \{E_2\}^{A_2} \longmapsto_\beta \{E_1 \bigtriangledown_\tau^{A_1,A_2} E_2\}^{A_1 \cup A_2} \quad \text{(M-SeqDis)}$$

$$\frac{M \longmapsto_\beta M'}{C[M] \longmapsto C[M']} \quad \text{(Ctx)}$$

A.2 Computations

$$\frac{M_1 \longmapsto M_2}{\vdash^{A_r} \sigma \triangleright M_1 \stackrel{\cdot}{\longmapsto} \sigma \triangleright M_2} \quad \text{(E-Ret)}$$

$$\frac{M_1 \longmapsto M_2}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = M_1\ \texttt{in}\ E \stackrel{\cdot}{\longmapsto} \sigma \triangleright \texttt{let}\ \{x\} = M_2\ \texttt{in}\ E} \quad \text{(E-Let1)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = \{v\}^A\ \texttt{in}\ E \stackrel{\cdot}{\longmapsto} \sigma \triangleright E[v/x]} \quad \text{(E-Let2)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = \{\texttt{halt}\}^A\ \texttt{in}\ E \stackrel{\cdot}{\longmapsto} \sigma \triangleright \texttt{halt}} \quad \text{(E-Let3)}$$

$$\frac{\vdash^A \cdot \triangleright E' \stackrel{\beta}{\longmapsto} \cdot \triangleright E''}{\vdash^{A_r} \cdot \triangleright \texttt{let}\ \{x\} = \{E'\}^A\ \texttt{in}\ E \stackrel{\beta}{\longmapsto} \cdot \triangleright \texttt{let}\ \{x\} = \{E''\}^A \texttt{in}\ E} \quad \text{(E-Let4)}$$

$$\frac{a \notin A}{\vdash^{A_r} a; \sigma \triangleright \texttt{let}\ \{x\} = \{E'\}^A\ \texttt{in}\ E \stackrel{\texttt{acc}(a)}{\longmapsto} \sigma \triangleright \texttt{let}\ \{x\} = \{E'\}^A\ \texttt{in}\ E} \quad \text{(E-Let5)}$$

$$\frac{a \in A}{\vdash^{A_r} a; \sigma \triangleright \texttt{let}\ \{x\} = \{E'\}^A\ \texttt{in}\ E \stackrel{\cdot}{\longmapsto} \sigma \triangleright \texttt{let}\ \{x\} = (a \triangleright E')^A\ \texttt{in}\ E} \quad \text{(E-Let6)}$$

$$\frac{\vdash^A a \triangleright E' \stackrel{\beta}{\longmapsto} a \triangleright E''}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = (a \triangleright E')^A\ \texttt{in}\ E \stackrel{\beta}{\longmapsto} \sigma \triangleright \texttt{let}\ \{x\} = (a \triangleright E'')^A\ \texttt{in}\ E} \quad \text{(E-Let7)}$$

$$\frac{\vdash^A a \triangleright E' \stackrel{\beta}{\longmapsto} \cdot \triangleright E''}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = (a \triangleright E')^A\ \texttt{in}\ E \stackrel{\beta}{\longmapsto} \sigma \triangleright \texttt{let}\ \{x\} = \{E''\}^A\ \texttt{in}\ E} \quad \text{(E-Let8)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \texttt{let}\ \{x\} = (a \triangleright v)^A\ \texttt{in}\ E \stackrel{\texttt{acc}(a)}{\longmapsto} \sigma \triangleright E[v/x]} \quad \text{(E-Let9)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{let}\ \{x\} = (a \triangleright \mathtt{halt})^A\ \mathtt{in}\ E \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-Let10)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{ok}; E \overset{\mathtt{acc}(a)}{\longmapsto} \sigma \triangleright E} \quad \text{(E-Acc)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{sup}; E \overset{\mathtt{sup}(a)}{\longmapsto} \sigma \triangleright E} \quad \text{(E-Sup)}$$

$$\frac{M_1 \longmapsto M_2}{\vdash^{A_r} \sigma \triangleright \mathtt{ins}(M_1); E \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{ins}(M_2); E} \quad \text{(E-Ins1)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{ins}(a); E \overset{\mathtt{ins}(a)}{\longmapsto} \sigma \triangleright E} \quad \text{(E-Ins2)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright (\mathtt{next} \to E_1 \mid \mathtt{done} \to E_2) \stackrel{\cdot}{\longmapsto} a; \sigma \triangleright E_1} \quad \text{(E-Next)}$$

$$\frac{a \notin A_r}{\vdash^{A_r} a; \sigma \triangleright (\mathtt{next} \to E_1 \mid \mathtt{done} \to E_2) \overset{\mathtt{acc}(a)}{\longmapsto} \sigma \triangleright (\mathtt{next} \to E_1 \mid \mathtt{done} \to E_2)} \quad \text{(E-Next-Skip)}$$

$$\frac{}{\vdash^{A_r} \cdot \triangleright (\mathtt{next} \to E_1 \mid \mathtt{done} \to E_2) \stackrel{\cdot}{\longmapsto} \cdot \triangleright E_2} \quad \text{(E-Done)}$$

$$\frac{a \in A_r}{\vdash^{A_r} a; \sigma \triangleright x\ \to E \stackrel{\cdot}{\longmapsto} a; \sigma \triangleright E[a/x]} \quad \text{(E-Bind)}$$

$$\frac{a \in A' \quad a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{acase}\ (\star\ \subseteq A')\ (E_1 \mid E_2) \stackrel{\cdot}{\longmapsto} a; \sigma \triangleright E_1} \quad \text{(E-Acase1)}$$

$$\frac{a \notin A' \quad a \in A_r}{\vdash^{A_r} a; \sigma \triangleright \mathtt{acase}\ (\star\ \subseteq A')\ (E_1 \mid E_2) \stackrel{\cdot}{\longmapsto} a; \sigma \triangleright E_2} \quad \text{(E-Acase2)}$$

$$\frac{M_1 \longmapsto M_2}{\begin{array}{l}\vdash^{A_r} \sigma \triangleright \mathtt{case}\ M_1\ (x \to E_1 \mid x \to E_2) \\ \qquad \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{case}\ M_2\ (x \to E_1 \mid x \to E_2)\end{array}} \quad \text{(E-CaseM)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{case}\ \mathtt{inl}_\tau(v)\ (x \to E_1 \mid x \to E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright E_1[v/x]} \quad \text{(E-CaseL)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{case}\ \mathtt{inr}_\tau(v)\ (x \to E_1 \mid x \to E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright E_2[v/x]} \quad \text{(E-CaseR)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright \mathtt{any} \stackrel{\cdot}{\longmapsto} \sigma \triangleright ()} \quad \text{(E-Any)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \|\beta\| \notin A_2 \text{ or } (E_2 = v \text{ and } \beta = \mathtt{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1' \wedge^{A_1,A_2} E_2)} \quad \text{(E-ParCon1)}$$

$$\frac{\vdash^{A_2} \sigma \triangleright E_2 \overset{\beta}{\longmapsto} \sigma' \triangleright E_2' \quad \|\beta\| \notin A_1 \text{ or } (E_1 = v \text{ and } \beta = \mathtt{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1 \wedge^{A_1,A_2} E_2')} \quad \text{(E-ParCon2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright E_1' \quad \vdash^{A_2} \sigma \triangleright E_2 \overset{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1,A_2} E_2) \overset{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright (E_1' \wedge^{A_1,A_2} E_2')} \quad \text{(E-ParCon3)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v_1 \wedge^{A_1,A_2} v_2) \overset{\cdot}{\longmapsto} \sigma \triangleright \langle v_1, v_2 \rangle} \quad \text{(E-ParCon4)}$$

$$\frac{E_1 = \mathtt{halt} \text{ or } E_2 = \mathtt{halt}}{\vdash^{A_r} \sigma \triangleright (E_1 \wedge^{A_1,A_2} E_2) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-ParCon5)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{sup}(a) \text{ or } \beta = \cdot}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1' \triangle^{A_1,A_2} E_2)} \quad \text{(E-SeqCon1)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \notin A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \sigma' \triangleright (E_1' \triangle^{A_1,A_2} E_2)} \quad \text{(E-SeqCon2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \in A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} E_2) \overset{\cdot}{\longmapsto} \sigma' \triangleright (E_1' \triangle^{A_1,A_2} (\beta \blacktriangleright E_2))} \quad \text{(E-SeqCon3)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \overset{\beta'}{\longmapsto} \|\beta\| \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright E_2)) \overset{\beta'}{\longmapsto} \sigma \triangleright (E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright E_2'))} \quad \text{(E-SeqCon4)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \overset{\beta'}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright E_2)) \overset{\beta''}{\longmapsto} \sigma \triangleright (E_1 \triangle^{A_1,A_2} E_2')} \quad \text{(E-SeqCon5)}$$

$$\text{where } \beta'' = \begin{cases} \cdot & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{ins}(a) & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{acc}(a) \\ \mathtt{sup}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{acc}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{acc}(a) \end{cases}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright v)) \overset{\beta}{\longmapsto} \sigma \triangleright (E_1 \triangle^{A_1,A_2} v)} \quad \text{(E-SeqCon6)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \triangle^{A_1,A_2} (\beta \blacktriangleright \mathtt{halt})) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-SeqCon7)}$$

$$\frac{E_2 \neq v \quad E_2 \neq \mathtt{halt}}{\vdash^{A_r} a; \sigma \triangleright (v \bigtriangleup^{A_1,A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright (v \bigtriangleup^{A_1,A_2} (\mathtt{acc}(a) \blacktriangleright E_2))} \quad \text{(E-SeqCon8)}$$

$$\frac{\vdash^{A_2} \cdot \triangleright E_2 \stackrel{\beta}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \cdot \triangleright (v \bigtriangleup^{A_1,A_2} E_2) \stackrel{\beta}{\longmapsto} \cdot \triangleright (v \bigtriangleup^{A_1,A_2} E_2')} \quad \text{(E-SeqCon9)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v_1 \bigtriangleup^{A_1,A_2} v_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \langle v_1, v_2 \rangle} \quad \text{(E-SeqCon10)}$$

$$\frac{E_1 = \mathtt{halt} \text{ or } E_2 = \mathtt{halt}}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangleup^{A_1,A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-SeqCon11)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \|\beta\| \notin A_2 \text{ or } (E_2 = \mathtt{halt} \text{ and } \beta = \mathtt{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \bigvee_\tau^{A_1,A_2} E_2) \stackrel{\beta}{\longmapsto} \sigma' \triangleright (E_1' \bigvee_\tau^{A_1,A_2} E_2)} \quad \text{(E-ParDis1)}$$

$$\frac{\vdash^{A_2} \sigma \triangleright E_2 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_2' \quad \|\beta\| \notin A_1 \text{ or } (E_1 = \mathtt{halt} \text{ and } \beta = \mathtt{acc}(a))}{\vdash^{A_r} \sigma \triangleright (E_1 \bigvee_\tau^{A_1,A_2} E_2) \stackrel{\beta}{\longmapsto} \sigma' \triangleright (E_1 \bigvee_\tau^{A_1,A_2} E_2')} \quad \text{(E-ParDis2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright E_1' \quad \vdash^{A_2} \sigma \triangleright E_2 \stackrel{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \bigvee_\tau^{A_1,A_2} E_2) \stackrel{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright (E_1' \bigvee_\tau^{A_1,A_2} E_2')} \quad \text{(E-ParDis3)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (\mathtt{halt} \bigvee_\tau^{A_1,A_2} \mathtt{halt}) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-ParDis4)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v \bigvee_\tau^{A_1,A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{inl}_\tau(v)} \quad \text{(E-ParDis5)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \bigvee_\tau^{A_1,A_2} v) \stackrel{\cdot}{\longmapsto} \sigma \triangleright \mathtt{inr}_\tau(v)} \quad \text{(E-ParDis6)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{sup}(a) \text{ or } \beta = \cdot}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma' \triangleright (E_1' \bigtriangledown_\tau^{A_1,A_2} E_2)} \quad \text{(E-SeqDis1)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \notin A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} E_2) \stackrel{\mathtt{acc}(a)}{\longmapsto} \sigma' \triangleright (E_1' \bigtriangledown_\tau^{A_1,A_2} E_2)} \quad \text{(E-SeqDis2)}$$

$$\frac{\vdash^{A_1} \sigma \triangleright E_1 \stackrel{\beta}{\longmapsto} \sigma' \triangleright E_1' \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a) \quad a \in A_2}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} E_2) \stackrel{\cdot}{\longmapsto} \sigma' \triangleright (E_1' \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2))} \quad \text{(E-SeqDis3)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \stackrel{\beta'}{\longmapsto} \|\beta\| \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2)) \stackrel{\beta'}{\longmapsto} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2'))} \quad \text{(E-SeqDis4)}$$

$$\frac{\vdash^{A_2} \|\beta\| \triangleright E_2 \overset{\beta'}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright E_2)) \overset{\beta''}{\longmapsto} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} E_2')} \quad \text{(E-SeqDis5)}$$

$$\text{where } \beta'' = \begin{cases} \cdot & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{ins}(a) & \text{if } \beta = \mathtt{ins}(a) \text{ and } \beta' = \mathtt{acc}(a) \\ \mathtt{sup}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{sup}(a) \\ \mathtt{acc}(a) & \text{if } \beta = \mathtt{acc}(a) \text{ and } \beta' = \mathtt{acc}(a) \end{cases}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright \mathtt{halt})) \overset{\beta}{\longmapsto} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} \mathtt{halt})} \quad \text{(E-SeqDis6)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} (\beta \blacktriangleright v)) \overset{\beta}{\longmapsto} \sigma \triangleright \mathtt{inr}_\tau(v)} \quad \text{(E-SeqDis7)}$$

$$\frac{E_2 \neq v \quad E_2 \neq \mathtt{halt}}{\vdash^{A_r} a; \sigma \triangleright (\mathtt{halt} \bigtriangledown_\tau^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \sigma \triangleright (\mathtt{halt} \bigtriangledown_\tau^{A_1,A_2} (\mathtt{acc}(a) \blacktriangleright E_2))} \quad \text{(E-SeqDis8)}$$

$$\frac{\vdash^{A_2} \cdot \triangleright E_2 \overset{\beta}{\longmapsto} \cdot \triangleright E_2'}{\vdash^{A_r} \cdot \triangleright (\mathtt{halt} \bigtriangledown_\tau^{A_1,A_2} E_2) \overset{\beta}{\longmapsto} \cdot \triangleright (\mathtt{halt} \bigtriangledown_\tau^{A_1,A_2} E_2')} \quad \text{(E-SeqDis9)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (\mathtt{halt} \bigtriangledown_\tau^{A_1,A_2} \mathtt{halt}) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{halt}} \quad \text{(E-SeqDis10)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (v \bigtriangledown_\tau^{A_1,A_2} E_2) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{inl}_\tau(v)} \quad \text{(E-SeqDis11)}$$

$$\frac{}{\vdash^{A_r} \sigma \triangleright (E_1 \bigtriangledown_\tau^{A_1,A_2} v) \overset{\cdot}{\longmapsto} \sigma \triangleright \mathtt{inr}_\tau(v)} \quad \text{(E-SeqDis12)}$$

A.3 System

$$\frac{a \notin A_r}{\mathsf{run}(a; \sigma, A_r, E) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma, A_r, E)} \quad \text{(R-Skip)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \quad \sigma = \cdot \text{ or } (\sigma = a'; \sigma'' \text{ and } a' \in A_r) \quad \beta = \cdot \text{ or } \beta = \mathtt{sup}(a)}{\mathsf{run}(\sigma, A_r, E) \overset{\cdot}{\hookrightarrow} \mathsf{run}(\sigma', A_r, E')} \quad \text{(R-Step1)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \quad \sigma = \cdot \text{ or } (\sigma = a'; \sigma'' \text{ and } a' \in A_r) \quad \beta = \mathtt{ins}(a) \text{ or } \beta = \mathtt{acc}(a)}{\mathsf{run}(\sigma, A_r, E) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma', A_r, E')} \quad \text{(R-Step2)}$$

$$\frac{}{\mathsf{run}(a; \sigma, A_r, v) \overset{a}{\hookrightarrow} \mathsf{run}(\sigma, A_r, v)} \quad \text{(R-Val1)}$$

$$\frac{}{\mathsf{run}(\cdot, A_r, v) \overset{\cdot}{\hookrightarrow} v} \quad \text{(R-Val2)}$$

$$\frac{}{\mathsf{run}(\sigma, A_r, \mathtt{halt}) \dot{\hookrightarrow} \mathtt{halt}} \quad \text{(R-Halt)}$$

B Static Semantics

B.1 Subtyping

$$\frac{A \subseteq A'}{\vdash \mathsf{act}(A) \leq \mathsf{act}(A')} \quad \text{(Sub-Act)}$$

$$\frac{\vdash \tau_1' \leq \tau_1 \quad \vdash \tau_2 \leq \tau_2'}{\vdash \tau_1 \to \tau_2 \leq \tau_1' \to \tau_2'} \quad \text{(Sub-Arrow)}$$

$$\frac{\vdash \tau_1 \leq \tau_1' \quad \vdash \tau_2 \leq \tau_2'}{\vdash \tau_1 \times \tau_2 \leq \tau_1' \times \tau_2'} \quad \text{(Sub-Pair)}$$

$$\frac{\vdash \tau_1 \leq \tau_1' \quad \vdash \tau_2 \leq \tau_2'}{\vdash \tau_1 + \tau_2 \leq \tau_1' + \tau_2'} \quad \text{(Sub-Sum)}$$

$$\frac{A_2 \subseteq A_2' \quad \vdash \tau \leq \tau'}{\vdash \mathcal{M}_{A_2}^{A_r}(\tau) \leq \mathcal{M}_{A_2'}^{A_r}(\tau')} \quad \text{(Sub-Monad)}$$

$$\frac{}{\vdash \tau \leq \tau} \quad \text{(Sub-Reflex)}$$

$$\frac{\vdash \tau_1 \leq \tau_2 \quad \vdash \tau_2 \leq \tau_3}{\vdash \tau_1 \leq \tau_3} \quad \text{(Sub-Trans)}$$

B.2 Ordinary Terms

$$\frac{}{\Gamma \vdash x : \Gamma(x)} \quad \text{(S-Var)}$$

$$\frac{}{\Gamma \vdash a : \mathsf{act}(\{a\})} \quad \text{(S-Act)}$$

$$\frac{\Gamma, f{:}\tau_1 \to \tau_2, x{:}\tau_1 \vdash M : \tau_2}{\Gamma \vdash \mathtt{fun}\ f{:}\tau_1 \to \tau_2\ (x).M : \tau_1 \to \tau_2} \quad \text{(S-Fun)}$$

$$\frac{\Gamma \vdash M_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash M_2 : \tau_1}{\Gamma \vdash M_1 M_2 : \tau_2} \quad \text{(S-App)}$$

$$\frac{}{\Gamma \vdash () : \mathsf{unit}} \quad \text{(S-Unit)}$$

$$\frac{\Gamma \vdash M_1 : \tau_1 \quad \Gamma \vdash M_2 : \tau_2}{\Gamma \vdash \langle M_1, M_2 \rangle : \tau_1 \times \tau_2} \quad \text{(S-Pair)}$$

$$\frac{\Gamma \vdash M : \tau_1 \times \tau_2}{\Gamma \vdash \pi_1\ M : \tau_1} \quad \text{(S-Proj1)}$$

$$\frac{\Gamma \vdash M : \tau_1 \times \tau_2}{\Gamma \vdash \pi_2\ M : \tau_2} \quad \text{(S-Proj2)}$$

$$\frac{\Gamma \vdash M : \tau_1}{\Gamma \vdash \mathtt{inl}_{\tau_1+\tau_2}(M) : \tau_1 + \tau_2} \quad \text{(S-Inl)}$$

$$\frac{\Gamma \vdash M : \tau_2}{\Gamma \vdash \mathtt{inr}_{\tau_1+\tau_2}(M) : \tau_1 + \tau_2} \quad \text{(S-Inr)}$$

$$\frac{\Gamma \vdash M_1 : \tau_1 + \tau_2 \quad \Gamma, x{:}\tau_1 \vdash M_2 : \tau \quad \Gamma, x{:}\tau_2 \vdash M_3 : \tau}{\Gamma \vdash \mathtt{case}\ M_1\ (x \to M_2 \mid x \to M_3) : \tau} \quad \text{(S-Case)}$$

$$\frac{\Gamma \vdash M : \tau' \quad \vdash \tau' \leq \tau}{\Gamma \vdash M : \tau} \quad \text{(S-Sub)}$$

B.3 Policies

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A_e}{\Gamma \vdash \{E\}^{A_r} : \mathcal{M}^{A_r}_{A_e}(\tau)} \quad \text{(S-Sus)}$$

$$\frac{}{\Gamma \vdash \top : \mathcal{M}^{\emptyset}_{\emptyset}(\mathsf{unit})} \quad \text{(S-Top)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2) \quad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \wedge M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2)} \quad \text{(S-ParCon)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)}{\Gamma \vdash M_1 \triangle M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2)} \quad \text{(S-SeqCon)}$$

$$\frac{}{\Gamma \vdash \bot : \mathcal{M}^{\emptyset}_{\emptyset}(\tau)} \quad \text{(S-Bot)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2) \quad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset}{\Gamma \vdash M_1 \vee_{\tau_1+\tau_2} M_2 : \mathcal{M}^{A_1 \cup A_2}_{A_3 \cup A_4}(\tau_1 + \tau_2)} \quad \text{(S-ParDis)}$$

$$\frac{\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1) \quad \Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)}{\Gamma \vdash M_1 \bigtriangledown_{\tau_1+\tau_2} M_2 : \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 + \tau_2)} \quad \text{(S-SeqDis)}$$

B.4 Computations

$$\frac{\Gamma \vdash M : \tau}{\Gamma; B \vdash^{A_r} M : \tau, \emptyset} \quad \text{(SE-Ret)}$$

$$\frac{\Gamma \vdash M : \mathcal{M}_{A_2}^{A_r'}(\tau') \quad \Gamma, x{:}\tau'; \diamond \vdash^{A_r} E : \tau, A \quad A_r' \subseteq A_r}{\Gamma; B \vdash^{A_r} \texttt{let}\ \{x\} = M \texttt{in}\ E : \tau, A \cup A_2} \quad \text{(SE-Let1)}$$

$$\frac{\cdot; \{a\} \vdash^{A} E_1 : \tau', A_1 \quad x{:}\tau'; \diamond \vdash^{A_r} E_2 : \tau, A_2 \quad A \subseteq A_r \quad a \in A}{\cdot; \{a\} \vdash^{A_r} \texttt{let}\ \{x\} = (a \triangleright E_1)^{A} \texttt{in}\ E_2 : \tau, A_1 \cup A_2} \quad \text{(SE-Let2)}$$

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} \texttt{ok}; E : \tau, A} \quad \text{(SE-Acc)}$$

$$\frac{\Gamma; \diamond \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} \texttt{sup}; E : \tau, A \cup B} \quad \text{(SE-Sup)}$$

$$\frac{\Gamma \vdash M : \mathsf{act}(A') \quad \Gamma; B \vdash^{A_r} E : \tau, A}{\Gamma; B \vdash^{A_r} \texttt{ins}(M); E : \tau, A \cup A'} \quad \text{(SE-Ins)}$$

$$\frac{\Gamma; A_r \vdash^{A_r} E_1 : \tau, A \quad \Gamma; \diamond \vdash^{A_r} E_2 : \tau, A}{\Gamma; B \vdash^{A_r} (\texttt{next} \to E_1 \mid \texttt{done} \to E_2) : \tau, A} \quad \text{(SE-Next)}$$

$$\frac{\Gamma, x{:}\mathsf{act}(B); B \vdash^{A_r} E : \tau, A \quad B \neq \diamond}{\Gamma; B \vdash^{A_r} x\ \to E : \tau, A} \quad \text{(SE-Bind)}$$

$$\frac{\begin{array}{c}\Gamma; B \cap_\diamond A' \vdash^{A_r} E_1 : \tau, A \qquad \Gamma; B \setminus_\diamond A' \vdash^{A_r} E_2 : \tau, A \\ A' \subseteq A_r \qquad B \neq \diamond\end{array}}{\Gamma; B \vdash^{A_r} \texttt{acase}\ (\star\ \subseteq A')\ (E_1 \mid E_2) : \tau, A} \quad \text{(SE-Acase)}$$

$$\frac{\begin{array}{c}\Gamma \vdash M : \tau_1 + \tau_2 \\ \Gamma, x{:}\tau_1; B \vdash^{A_r} E_1 : \tau, A \qquad \Gamma, x{:}\tau_2; B \vdash^{A_r} E_2 : \tau, A\end{array}}{\Gamma; B \vdash^{A_r} \texttt{case}\ M\ (x \to E_1 \mid x \to E_2) : \tau, A} \quad \text{(SE-Case)}$$

$$\frac{}{\Gamma; B \vdash^{A_r} \texttt{any} : \mathsf{unit}, \emptyset} \quad \text{(SE-Any)}$$

$$\frac{\begin{array}{c}\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \Gamma; B \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4 \\ A_1 \cap A_4 = A_2 \cap A_3 = \emptyset \qquad A_1 \cup A_2 = A_r\end{array}}{\Gamma; B \vdash^{A_r} E_1 \wedge^{A_1, A_2} E_2 : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-ParCon)}$$

$$\frac{\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \quad \Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4 \quad A_1 \cup A_2 = A_r}{\Gamma; B \vdash^{A_r} E_1 \triangle^{A_1, A_2} E_2 : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon1)}$$

$$\frac{\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \qquad A_1 \cup A_2 = A_r \qquad a \in A_2 \qquad a \in A_3}{\cdot; B \vdash^{A_r} E_1 \triangle^{A_1,A_2} (\texttt{ins}(a) \blacktriangleright E_2) : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon2)}$$

$$\frac{\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \qquad A_1 \cup A_2 = A_r \qquad a \in A_2}{\cdot; B \vdash^{A_r} E_1 \triangle^{A_1,A_2} (\texttt{acc}(a) \blacktriangleright E_2) : \tau_1 \times \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqCon3)}$$

$$\frac{}{\Gamma; B \vdash^{A_r} \texttt{halt} : \tau, \emptyset} \quad \text{(SE-Halt)}$$

$$\frac{\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \Gamma; B \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4 \qquad A_1 \cap A_4 = A_2 \cap A_3 = \emptyset \qquad A_1 \cup A_2 = A_r}{\Gamma; B \vdash^{A_r} E_1 \vee^{A_1,A_2}_{\tau_1+\tau_2} E_2 : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-ParDis)}$$

$$\frac{\Gamma; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \quad \Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4 \quad A_1 \cup A_2 = A_r}{\Gamma; B \vdash^{A_r} E_1 \bigtriangledown^{A_1,A_2}_{\tau_1+\tau_2} E_2 : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis1)}$$

$$\frac{\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \qquad A_1 \cup A_2 = A_r \qquad a \in A_2 \qquad a \in A_3}{\cdot; B \vdash^{A_r} E_1 \bigtriangledown^{A_1,A_2}_{\tau_1+\tau_2} (\texttt{ins}(a) \blacktriangleright E_2) : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis2)}$$

$$\frac{\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3 \qquad \cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4 \qquad A_1 \cup A_2 = A_r \qquad a \in A_2}{\cdot; B \vdash^{A_r} E_1 \bigtriangledown^{A_1,A_2}_{\tau_1+\tau_2} (\texttt{acc}(a) \blacktriangleright E_2) : \tau_1 + \tau_2, A_3 \cup A_4} \quad \text{(SE-SeqDis3)}$$

$$\frac{\Gamma; B' \vdash^{A_r} E : \tau', A' \quad (B' = \diamond \ \texttt{or} \ B \subseteq B') \quad \vdash \tau' \leq \tau \quad A' \subseteq A}{\Gamma; B \vdash^{A_r} E : \tau, A} \quad \text{(SE-Sub)}$$

B.5 System

$$\frac{\vdash \sigma : B \quad \cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A}{\vdash^{A_r} \sigma \triangleright E \ \texttt{ok}} \quad \text{(S-Ok)}$$

$$\frac{}{\vdash \cdot : \diamond} \quad \text{(Seq-Empty)}$$

$$\frac{}{\vdash a; \sigma : \{a\}} \quad \text{(Seq-Act)}$$

$$\frac{\vdash^{A_r} \sigma \triangleright E \ \texttt{ok}}{\vdash \mathsf{run}(\sigma, A_r, E) \ \texttt{ok}} \quad \text{(Run-Ok)}$$

$$\frac{\vdash v : \tau}{\vdash v \ \texttt{ok}} \quad \text{(Val-Ok)}$$

$$\frac{}{\vdash \texttt{halt ok}} \qquad \text{(Halt-Ok)}$$

C Proof of Safety

We define new notation in order to reduce the verbosity of our lemmas and proofs. First, we say that B is *first-action compatible* with B' (written $B \precsim B'$ or $B' \succsim B$) when either $B' = \diamond$ or $B \subseteq B'$. Intuitively, this first-action compatibility operator behaves like the standard subset operator, except that $\diamond$ is considered to be larger than any set; therefore, anything (including $\diamond$ itself) is first-action compatible with $\diamond$. This ensures that B contains no less knowledge of the next possible action than B' when $B \precsim B'$. We use first-action compatibility to define subtyping compatibility as follows: $(B', \tau', A') \ll (B, \tau, A)$ indicates that $B \precsim B'$, $\vdash \tau' \leq \tau$, and $A' \subseteq A$. This states that computation subtyping is contravariant in first-action sets but covariant in type and effect sets. Thus, by rule (SE-Sub), if $(B', \tau', A') \ll (B, \tau, A)$ and $\Gamma; B' \vdash^{A_r} E : \tau', A'$ then $\Gamma; B \vdash^{A_r} E : \tau, A$.

We state the main lemmas in our proof of type safety and provide the proof technique for each lemma. For the lemmas with non-trivial proofs, we also provide proofs for a selection of interesting cases.

Lemma 1 (Substitution)

1. *If* $\Gamma, x : \tau' \vdash M : \tau$ *and* $\Gamma \vdash M' : \tau'$ *then* $\Gamma \vdash M[M'/x] : \tau$, *and*
2. *If* $\Gamma, x : \tau'; B \vdash^{A_r} E : \tau, A$ *and* $\Gamma \vdash M' : \tau'$ *then* $\Gamma; B \vdash^{A_r} E[M'/x] : \tau, A$.

Proof: By mutual induction on the typing derivations of $\Gamma, x : \tau' \vdash M : \tau$ and $\Gamma, x : \tau'; B \vdash^{A_r} E : \tau, A$. ■

Lemma 2 (Inversion of Subtyping)
If $\vdash \tau \leq \tau'$ *then*

1. $\tau' = \mathsf{unit}$ *if and only if* $\tau = \mathsf{unit}$
2. *If* $\tau' = \tau_1' \times \tau_2'$ *then* $\tau = \tau_1 \times \tau_2$, $\vdash \tau_1 \leq \tau_1'$, *and* $\vdash \tau_2 \leq \tau_2'$
3. *If* $\tau = \tau_1 \times \tau_2$ *then* $\tau' = \tau_1' \times \tau_2'$, $\vdash \tau_1 \leq \tau_1'$, *and* $\vdash \tau_2 \leq \tau_2'$
4. *If* $\tau' = \tau_1' + \tau_2'$ *then* $\tau = \tau_1 + \tau_2$, $\vdash \tau_1 \leq \tau_1'$, *and* $\vdash \tau_2 \leq \tau_2'$
5. *If* $\tau = \tau_1 + \tau_2$ *then* $\tau' = \tau_1' + \tau_2'$, $\vdash \tau_1 \leq \tau_1'$, *and* $\vdash \tau_2 \leq \tau_2'$
6. *If* $\tau' = \tau_1' \to \tau_2'$ *then* $\tau = \tau_1 \to \tau_2$, $\vdash \tau_1' \leq \tau_1$, *and* $\vdash \tau_2 \leq \tau_2'$
7. *If* $\tau = \tau_1 \to \tau_2$ *then* $\tau' = \tau_1' \to \tau_2'$, $\vdash \tau_1' \leq \tau_1$, *and* $\vdash \tau_2 \leq \tau_2'$
8. *If* $\tau' = \mathcal{M}^{A_r}_{A_e'}(\tau_1')$ *then* $\tau = \mathcal{M}^{A_r}_{A_e}(\tau_1)$, $\vdash \tau_1 \leq \tau_1'$, *and* $A_e \subseteq A_e'$
9. *If* $\tau = \mathcal{M}^{A_r}_{A_e}(\tau_1)$ *then* $\tau' = \mathcal{M}^{A_r}_{A_e'}(\tau_1')$, $\vdash \tau_1 \leq \tau_1'$, *and* $A_e \subseteq A_e'$
10. *If* $\tau' = \mathsf{act}(A')$ *then* $\tau = \mathsf{act}(A)$ *and* $A \subseteq A'$
11. *If* $\tau = \mathsf{act}(A)$ *then* $\tau' = \mathsf{act}(A')$ *and* $A \subseteq A'$

Proof: By induction on the derivation of $\vdash \tau \leq \tau'$. The interesting proof cases are similar, so we show only case 8. The final rule deriving $\vdash \tau \leq \mathcal{M}^{A_r}_{A'_e}(\tau'_1)$ is either (Sub-Monad), (Sub-Reflex), or (Sub-Trans), with the result being immediate in the first two cases.

Case (Sub-Trans):

By assumption,

$$\tau' = \mathcal{M}^{A_r}_{A'_e}(\tau'_1) \quad (1)$$
$$\vdash \tau \leq \tau'' \quad (2)$$
$$\vdash \tau'' \leq \tau' \quad (3)$$

By Inductive Hypothesis (IH), (1), and (3),

$$\tau'' = \mathcal{M}^{A_r}_{A''_e}(\tau''_1) \quad (4)$$
$$\vdash \tau''_1 \leq \tau'_1 \quad (5)$$
$$A''_e \subseteq A'_e \quad (6)$$

By IH, (2), and (4),

$$\tau = \mathcal{M}^{A_r}_{A_e}(\tau_1) \quad (7)$$
$$\vdash \tau_1 \leq \tau''_1 \quad (8)$$
$$A_e \subseteq A''_e \quad (9)$$

By (5), (8), and rule (Sub-Trans),

$$\vdash \tau_1 \leq \tau'_1 \quad (10)$$

By (6) and (9),

$$A_e \subseteq A'_e \quad (11)$$

Result is from (7), (10), and (11). ■

Lemma 3 (Canonical Forms: Values)
If $\cdot \vdash v : \tau$ *then*

1. *If* $\tau = \mathsf{unit}$ *then* $v = ()$
2. *If* $\tau = \mathsf{act}(A)$ *then* $v = a$ *and* $a \in A$.
3. *If* $\tau = \tau_1 \to \tau_2$ *then*
   - $v = \mathtt{fun}\ f : \tau'_1 \to \tau'_2(x).M$
   - $\vdash \tau_1 \leq \tau'_1$
   - $\vdash \tau'_2 \leq \tau_2$
4. *If* $\tau = \tau_1 \times \tau_2$ *then*
   - $v = \langle v_1, v_2 \rangle$
   - $\cdot \vdash v_1 : \tau_1$
   - $\cdot \vdash v_2 : \tau_2$
5. *If* $\tau = \tau_1 + \tau_2$ *then either*
   - $v = \mathtt{inl}_{\tau'_1+\tau'_2}(v_1)$
   - $\cdot \vdash v_1 : \tau'_1$
   - $\vdash \tau'_1 \leq \tau_1$
   - $\vdash \tau'_2 \leq \tau_2$

   or

- $v = \mathtt{inr}_{\tau_1'+\tau_2'}(v_2)$
- $\cdot \vdash v_2 : \tau_2'$
- $\vdash \tau_1' \leq \tau_1$
- $\vdash \tau_2' \leq \tau_2$

6. *If* $\tau = \mathcal{M}^{A_r}_{A_e}(\tau_1)$ *then*
   - $v = \{E\}^{A_r}$
   - $\cdot;\diamond \vdash^{A_r} E : \tau_1', A_e'$
   - $\vdash \mathcal{M}^{A_r}_{A_e'}(\tau_1') \leq \mathcal{M}^{A_r}_{A_e}(\tau_1)$

Proof: By induction on the derivation of $\cdot \vdash v : \tau$. We show the interesting case for monadic types. The final rule deriving $\cdot \vdash v : \mathcal{M}^{A_r}_{A_e}(\tau_1)$ is either (S-Sus) (from which the result is immediate) or (S-Sub).

Case (S-Sub):

By assumption,

$\cdot \vdash v : \tau'$ (1)

$\vdash \tau' \leq \tau$ (2)

$\tau = \mathcal{M}^{A_r}_{A_e}(\tau_1)$ (3)

By Lemma 2, (2), and (3),

$\tau' = \mathcal{M}^{A_r}_{A_e'}(\tau_1')$ (4)

$\vdash \tau_1' \leq \tau_1$ (5)

$A_e' \subseteq A_e$ (6)

By IH, (1), and (4),

$v = \{E\}^{A_r}$ (7)

$\cdot;\diamond \vdash^{A_r} E : \tau_1'', A_e''$ (8)

$\vdash \mathcal{M}^{A_r}_{A_e''}(\tau_1'') \leq \mathcal{M}^{A_r}_{A_e'}(\tau_1')$ (9)

By Lemma 2 and (9),

$\vdash \tau_1'' \leq \tau_1'$ (10)

$A_e'' \subseteq A_e'$ (11)

By (8), (10), (11), and rule (SE-Sub),

$\cdot;\diamond \vdash^{A_r} E : \tau_1', A_e'$ (12)

By (5), (6), and rule (Sub-Monad),

$\vdash \mathcal{M}^{A_r}_{A_e'}(\tau_1') \leq \mathcal{M}^{A_r}_{A_e}(\tau_1)$ (13)

Result is from (7), (12), and (13). ■

Lemma 4 (Inversion of Term Typing)
If $\Gamma \vdash M : \tau$ *then*

1. *If* $M = x$ *then*
   - $\Gamma(x) = \tau'$
   - $\tau' \leq \tau$
2. *If* $M = a$ *then*
   - $\tau = \mathsf{act}(A)$
   - $a \in A$

3. *If* $M = ()$ *then* $\tau = \mathsf{unit}$
4. *If* $M = M_1 M_2$ *then*
   - $\Gamma \vdash M_1 : \tau_1 \to \tau$
   - $\Gamma \vdash M_2 : \tau_1$
5. *If* $M = \langle M_1, M_2 \rangle$ *then*
   - $\tau = \tau_1 \times \tau_2$
   - $\Gamma \vdash M_1 : \tau_1$
   - $\Gamma \vdash M_2 : \tau_2$
6. *If* $M = \mathtt{fun}\ f : \tau_1 \to \tau_2(x).M'$ *then*
   - $\Gamma, f : \tau_1 \to \tau_2, x : \tau_1 \vdash M' : \tau_2$
   - $\vdash \tau_1 \to \tau_2 \leq \tau$
7. *If* $M = \pi_1\ M_1$ *then* $\Gamma \vdash M_1 : \tau \times \tau_2$
8. *If* $M = \pi_2\ M_2$ *then* $\Gamma \vdash M_2 : \tau_1 \times \tau$
9. *If* $M = \mathtt{inl}_{\tau_1+\tau_2}(M')$ *then*
   - $\tau = \tau_1' + \tau_2'$
   - $\Gamma \vdash M' : \tau_1$
   - $\vdash \tau_1 \leq \tau_1'$
   - $\vdash \tau_2 \leq \tau_2'$
10. *If* $M = \mathtt{inr}_{\tau_1+\tau_2}(M')$ *then*
    - $\tau = \tau_1' + \tau_2'$
    - $\Gamma \vdash M' : \tau_2$
    - $\vdash \tau_1 \leq \tau_1'$
    - $\vdash \tau_2 \leq \tau_2'$
11. *If* $M = \mathtt{case}\ M_1\ (x \to M_2 \mid x \to M_3)$ *then*
    - $\Gamma \vdash M_1 : \tau_1 + \tau_2$
    - $\Gamma, x : \tau_1 \vdash M_2 : \tau$
    - $\Gamma, x : \tau_2 \vdash M_3 : \tau$
12. *If* $M = \{E\}^{A_r}$ *then*
    - $\tau = \mathcal{M}^{A_r}_{A_e}(\tau'')$
    - $\Gamma; \diamond \vdash^{A_r} E : \tau', A_e'$
    - $\vdash \mathcal{M}^{A_r}_{A_e'}(\tau') \leq \mathcal{M}^{A_r}_{A_e}(\tau'')$
13. *If* $M = \top$ *then* $\tau = \mathcal{M}^{\emptyset}_{A_e}(\mathsf{unit})$
14. *If* $M = M_1 \wedge M_2$ *then*
    - $\vdash \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2) \leq \tau$
    - $\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1)$

- $\Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)$
- $A_1 \cap A_4 = A_2 \cap A_3 = \emptyset$

15. *If* $M = M_1 \triangle M_2$ *then*
    - $\vdash \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 \times \tau_2) \leq \tau$
    - $\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1)$
    - $\Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)$
16. *If* $M = \bot$ *then* $\tau = \mathcal{M}^{\emptyset}_{A_e}(\tau')$
17. *If* $M = M_1 \vee_{\tau_1+\tau_2} M_2$ *then*
    - $\vdash \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 + \tau_2) \leq \tau$
    - $\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1)$
    - $\Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)$
    - $A_1 \cap A_4 = A_2 \cap A_3 = \emptyset$
18. *If* $M = M_1 \bigtriangledown_{\tau_1+\tau_2} M_2$ *then*
    - $\vdash \mathcal{M}^{A_1 \cup A_3}_{A_2 \cup A_4}(\tau_1 + \tau_2) \leq \tau$
    - $\Gamma \vdash M_1 : \mathcal{M}^{A_1}_{A_2}(\tau_1)$
    - $\Gamma \vdash M_2 : \mathcal{M}^{A_3}_{A_4}(\tau_2)$

Proof: By induction on the derivation of $\Gamma \vdash M : \tau$. In every case, the final rule deriving $\Gamma \vdash M : \tau$ is either some base case, from which the result is immediate, or (S-Sub), from which the result is also immediate either directly by induction or by induction and the transitivity of the subtyping relation. ■

Definition 5 (Context and Hole Typing)
To facilitate the proof of preservation for terms, we define what it means for a context C *to be well-typed. We write* C^τ *for a context with a hole of type* τ*. More formally,*

$$\frac{[\,] : \tau \vdash C : \tau'}{\vdash C^\tau : \tau'} \qquad \text{(S-Context)}$$

Lemma 6 (Context Substitution)
If $\vdash C^\tau : \tau'$ *and* $\vdash M : \tau$ *then* $\vdash C[M] : \tau'$*.*

Proof: Immediate by inversion of (S-Context) and the Substitution Lemma (Lemma 1). ■

Lemma 7 (Context Typing)
If $\vdash C[M] : \tau$ *then there exists a* τ' *such that* $\vdash C^{\tau'} : \tau$ *and* $\vdash M : \tau'$*.*

Proof: By induction on the structure of C. We show two cases; the others are similar to the latter case.

Case C=[ ]:

By assumption,

$\vdash C[M] : \tau$ (1)

$C = [\,]$ (2)

By (1) and (2),

$[\,] : \tau \vdash C : \tau$ (3)

$\vdash M : \tau$ (4)

By (S-Context) and (3),

$\vdash C^{\tau} : \tau$ (5)

Case C= $\langle C', M' \rangle$:

By assumption,

$\vdash C[M] : \tau$ (1)

$C = \langle C', M' \rangle$ (2)

By (1) and (2),

$\vdash \langle C'[M], M' \rangle : \tau$ (3)

By Lemma 4 and (3),

$\tau = \tau_1 \times \tau_2$ (4)

$\vdash C'[M] : \tau_1$ (5)

$\vdash M' : \tau_2$ (6)

By IH and (5),

$\vdash C'^{\tau'} : \tau_1$ (7)

$\vdash M : \tau'$ (8)

By inversion of (S-Context) and (7),

$[\,] : \tau' \vdash C' : \tau_1$ (9)

By (2), (6), (9), and rule (S-Pair),

$[\,] : \tau' \vdash C : \tau_1 \times \tau_2$ (10)

By (4) and (10),

$[\,] : \tau' \vdash C : \tau$ (11)

By (11) and rule (S-Context),

$\vdash C^{\tau'} : \tau$ (12)

Result is from (8) and (12). ■

Lemma 8 (Preservation: $\longmapsto_\beta$)
If $\vdash M : \tau$ *and* $M \longmapsto_\beta M'$ *then* $\vdash M' : \tau$.

Proof: By induction on the derivation of $M \longmapsto_\beta M'$, using Inversion of Term Typing (Lemma 4). ■

Lemma 9 (Preservation: Terms)
If $\vdash M_1 : \tau$ *and* $M_1 \longmapsto M_2$ *then* $\vdash M_2 : \tau$.

Proof: By rule (Ctx) and Lemmas 6, 7, and 8.

By assumption,

$\vdash M_1 : \tau$ (1)

$M_1 \longmapsto M_2$ (2)

By (2) and rule (Ctx),

$M_1 = C[M_1']$ (3)

$M_2 = C[M_2']$ (4)
$M_1' \longmapsto M_2'$ (5)
By (1) and (3),
$C[M_1'] : \tau$ (6)
By Lemma 7 and (6),
$\vdash C^{\tau'} : \tau$ (7)
$\vdash M_1' : \tau'$ (8)
By Lemma 8, (8), and (5),
$\vdash M_2' : \tau'$ (9)
By Lemma 6, (7), and (9),
$\vdash C[M_2'] : \tau$ (10)
By (4) and (10),
$\vdash M_2 : \tau$ (11)

■

Lemma 10 (Progress: Terms and Redexes)
If $\vdash M : \tau$ *then either*

1. $M = C[R]$, *where* R *is a redex of one of the following forms*
   - $\texttt{fun}\ f{:}\tau(x).M\ v$
   - $\pi_1\ \langle v_1, v_2 \rangle$
   - $\pi_2\ \langle v_1, v_2 \rangle$
   - $\texttt{case}\ \texttt{inl}_\tau(v)\ (x \to M_1 \mid x \to M_2)$
   - $\texttt{case}\ \texttt{inr}_\tau(v)\ (x \to M_1 \mid x \to M_2)$
   - $\top$
   - $\{E_1\}^{A_1} \wedge \{E_2\}^{A_2}$
   - $\{E_1\}^{A_1} \triangle \{E_2\}^{A_2}$
   - $\bot$
   - $\{E_1\}^{A_1} \vee_\tau \{E_2\}^{A_2}$
   - $\{E_1\}^{A_1} \bigtriangledown_\tau \{E_2\}^{A_2}$

Or

2. $M = v$

Proof: By induction on the derivation of $\vdash M : \tau$, using the Canonical Forms Lemma (Lemma 3). ■

Lemma 11 (Progress: Terms)
If $\cdot \vdash M : \tau$ *then*

1. $M = v$, *or*
2. $M \longmapsto M'$

Proof: Immediate by Lemma 10 above and rule (Ctx). ■

Lemma 12 (Transitivity of Subtyping Compatibility)
If $(B'', \tau'', A'') \ll (B', \tau', A')$ *and* $(B', \tau', A') \ll (B, \tau, A)$ *then* $(B'', \tau'', A'') \ll (B, \tau, A)$.

Proof: By transitivity of the relations defining subtyping compatibility. ■

Lemma 13 (Inversion of Computation Typing)
If $\Gamma; B \vdash^{A_r} E : \tau, A$ *then*

1. *If* $E = M$ *then* $\Gamma \vdash M : \tau$
2. *If* $E = \texttt{let}\ \{x\} = M\ \texttt{in}\ E'$ *then*
   - $\Gamma \vdash M : \mathcal{M}_{A_2}^{A'_r}(\tau'')$
   - $\Gamma, x : \tau''; \diamond \vdash^{A_r} E' : \tau', A_1$
   - $A'_r \subseteq A_r$
   - $(B, \tau', A_1 \cup A_2) \ll (B, \tau, A)$
3. *If* $E = \texttt{let}\ \{x\} = (a \triangleright E_1)^{A'}\ \texttt{in}\ E_2$ *then*
   - $\cdot; \{a\} \vdash^{A'} E_1 : \tau'', A_1$
   - $a \in A'$
   - $x : \tau''; \diamond \vdash^{A_r} E_2 : \tau', A_2$
   - $A' \subseteq A_r$
   - $(B, \tau', A_1 \cup A_2) \ll (B, \tau, A)$
4. *If* $E = \texttt{ok}; E'$ *then*
   - $\Gamma; \diamond \vdash^{A_r} E' : \tau', A'$
   - $B' \neq \diamond$
   - $(B', \tau', A') \ll (B, \tau, A)$
5. *If* $E = \texttt{sup}; E'$ *then*
   - $\Gamma; \diamond \vdash^{A_r} E' : \tau', A'$
   - $B' \neq \diamond$
   - $(B', \tau', A' \cup B') \ll (B, \tau, A)$
6. *If* $E = \texttt{ins}(M); E'$ *then*
   - $\Gamma \vdash M : \mathsf{act}(A'')$
   - $\Gamma; B' \vdash^{A_r} E' : \tau', A'$
   - $(B', \tau', A' \cup A'') \ll (B, \tau, A)$
7. *If* $E = (\texttt{next} \to E_1 \mid \texttt{done} \to E_2)$ *then*
   - $\Gamma; A_r \vdash^{A_r} E_1 : \tau', A'$
   - $\Gamma; \diamond \vdash^{A_r} E_2 : \tau', A'$
   - $(B, \tau', A') \ll (B, \tau, A)$
8. *If* $E = x \to E'$ *then*

- $\Gamma, x : \mathsf{act}(B'); B' \vdash^{A_r} E' : \tau', A'$
- $B' \neq \diamond$
- $(B', \tau', A') \ll (B, \tau, A)$

9. *If* $E = \mathsf{acase}\ (\star \subseteq A_1)\ (E_1 \mid E_2)$ *then*
   - $\Gamma; B' \cap_\diamond A_1 \vdash^{A_r} E_1 : \tau', A'$
   - $\Gamma; B' \setminus_\diamond A_1 \vdash^{A_r} E_2 : \tau', A'$
   - $A_1 \subseteq A_r$
   - $B' \neq \diamond$
   - $(B', \tau', A') \ll (B, \tau, A)$
10. *If* $E = \mathsf{case}\ M\ (x \to E_1 \mid x \to E_2)$ *then*
   - $\Gamma \vdash M : \tau_1 + \tau_2$
   - $\Gamma, x : \tau_1; B' \vdash^{A_r} E_1 : \tau', A'$
   - $\Gamma, x : \tau_2; B' \vdash^{A_r} E_2 : \tau', A'$
   - $(B', \tau', A') \ll (B, \tau, A)$
11. *If* $E = \mathsf{any}$ *then* $\tau = \mathsf{unit}$
12. *If* $E = E_1 \wedge^{A_1, A_2} E_2$ *then*
   - $\Gamma; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
   - $\Gamma; B' \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4$
   - $A_1 \cap A_4 = A_2 \cap A_3 = \emptyset$
   - $A_1 \cup A_2 = A_r$
   - $(B', \tau_1 \times \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
13. *If* $E = E_1 \triangle^{A_1, A_2} E_2$ *then*
   - $\Gamma; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
   - $\Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4$
   - $A_1 \cup A_2 = A_r$
   - $(B', \tau_1 \times \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
14. *If* $E = E_1 \triangle^{A_1, A_2} (\mathtt{ins}(a) \blacktriangleright E_2)$ *then*
   - $\cdot; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
   - $\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$
   - $A_1 \cup A_2 = A_r$
   - $a \in A_2$
   - $a \in A_3$
   - $(B', \tau_1 \times \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
15. *If* $E = E_1 \triangle^{A_1, A_2} (\mathtt{acc}(a) \blacktriangleright E_2)$ *then*
   - $\cdot; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
   - $\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$

- $A_1 \cup A_2 = A_r$
- $a \in A_2$
- $(B', \tau_1 \times \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$

16. *If* $E = E_1 \vee^{A_1, A_2}_{\tau_1 + \tau_2} E_2$ *then*
    - $\Gamma; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
    - $\Gamma; B' \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4$
    - $A_1 \cap A_4 = A_2 \cap A_3 = \emptyset$
    - $A_1 \cup A_2 = A_r$
    - $(B', \tau_1 + \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
17. *If* $E = E_1 \bigtriangledown^{A_1, A_2}_{\tau_1 + \tau_2} E_2$ *then*
    - $\Gamma; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
    - $\Gamma; \diamond \vdash^{A_2} E_2 : \tau_2, A_4$
    - $A_1 \cup A_2 = A_r$
    - $(B', \tau_1 + \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
18. *If* $E = E_1 \bigtriangledown^{A_1, A_2}_{\tau_1 + \tau_2} (\texttt{ins}(a) \blacktriangleright E_2)$ *then*
    - $\cdot; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
    - $\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$
    - $A_1 \cup A_2 = A_r$
    - $a \in A_2$
    - $a \in A_3$
    - $(B', \tau_1 + \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$
19. *If* $E = E_1 \bigtriangledown^{A_1, A_2}_{\tau_1 + \tau_2} (\texttt{acc}(a) \blacktriangleright E_2)$ *then*
    - $\cdot; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$
    - $\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$
    - $A_1 \cup A_2 = A_r$
    - $a \in A_2$
    - $(B', \tau_1 + \tau_2, A_3 \cup A_4) \ll (B, \tau, A)$

Proof: By induction on the derivation of $\Gamma; B \vdash^{A_r} E : \tau, A$. In every case, the final rule deriving $\Gamma; B \vdash^{A_r} E : \tau, A$ is either some base case, from which the result is immediate, or (SE-Sub), from which the result is also immediate directly by induction, Lemma 12, and the definition of subtyping compatibility. ■

Lemma 14 (Forms of Streams in Evaluation)
If $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$ *then either*

1. $\sigma = \sigma'$, *or*
2. $\sigma = a; \sigma'$

Proof: By induction on the single-step operational semantics for computations. ■

Lemma 15 (Canonical Forms: Streams)
If $\vdash \sigma : B$ *then either*

1. $B = \{a\}$ *and* $\sigma = a; \sigma'$, *or*
2. $B = \diamond$ *and* $\sigma = \cdot$

Proof: Immediate by inspection of rules (Seq-Empty) and (Seq-Act). ■

Lemma 16 (Compatibility with Subsets)
If $A \subseteq A_r$ *and* $B \sqcup A_r \precsim B'$ *then* $B \sqcup A \precsim B' \sqcup A$.

Proof: There are two cases to consider. When $B \subseteq A_r$, $B \sqcup A_r \precsim B'$ reduces to $B \precsim B'$, from which the result is obtained easily. In the other case, $B \not\subseteq A_r$, so $B \sqcup A_r \precsim B'$ reduces to $\diamond \precsim B'$, implying that $B' = \diamond$ and the result is again immediate. ■

Lemma 17 (Inversion of System Typing)
If $\vdash^{A_r} \sigma \triangleright E$ `ok` *then* $\vdash \sigma : B$ *and* $\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$.

Proof: By examination of (S-Ok). ■

Lemma 18 (Effects are in Effect Set)
If

- $\vdash^{A_r} \sigma \triangleright E \stackrel{\beta}{\longmapsto} \sigma' \triangleright E'$
- $\vdash \sigma : B$
- $\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$
- $\beta = \mathtt{ins}(a)$ *or* $\beta = \mathtt{sup}(a)$

then $a \in A$.

Proof: By induction on the single-step operational semantics for computations. Most of the rules are uninteresting because β cannot be $\mathtt{ins}(a)$ or $\mathtt{sup}(a)$. We show two of the interesting cases; the others are very similar.

Case (E-Sup):
By assumption,

$$E = \mathtt{sup}; E' \quad (1)$$
$$\vdash \sigma : B \quad (2)$$
$$\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A \quad (3)$$
$$a \in A_r \quad (4)$$
$$\sigma = a; \sigma' \quad (5)$$

By Lemma 13, (3), and (1),

$$B' \neq \diamond \quad (6)$$
$$(B', \tau', A' \cup B') \ll (B \sqcup A_r, \tau, A) \quad (7)$$

By Lemma 15, (2), and (5),

$B = \{a\}$ (8)

By (4) and (8),

$B \sqcup A_r = \{a\}$ (9)

By (6), (7), and (9),

$\{a\} \subseteq B'$ (10)

$A' \cup B' \subseteq A$ (11)

By (10) and (11),

$a \in A$ (12)

Case (E-SeqCon5):

By assumption,

$E = (E_1 \triangle^{A_1, A_2} (\beta'' \blacktriangleright E_2))$ (1)

$\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$ (2)

$\vdash^{A_2} \|\beta''\| \triangleright E_2 \overset{\beta'}{\longmapsto} \cdot \triangleright E_2'$ (3)

$\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma \triangleright (E_1 \triangle^{A_1, A_2} E_2')$ (4)

$\beta = \mathtt{ins}(a)$ *or* $\beta = \mathtt{sup}(a)$ (5)

By (4), (5), and the definition of β in rule (E-SeqCon5),

$\beta'' = \mathtt{ins}(a)$ *or* $(\beta' = \mathtt{sup}(a)$ *and* $\beta'' = \mathtt{acc}(a))$ (6)

By Lemma 13, (1), (2), and the definition of subtyping compatibility,

$(\beta'' = \mathtt{ins}(a)) \Rightarrow$

$a \in A_3$ (7)

$A_3 \cup A_4 \subseteq A$ (8)

By (7) and (8),

$(\beta'' = \mathtt{ins}(a)) \Rightarrow a \in A$ (9)

By Lemma 13, (1), (2), and the definition of subtyping compatibility,

$\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$ (10)

$a \in A_2$ (11)

$A_3 \cup A_4 \subseteq A$ (12)

By (10) and (11),

$(\beta' = \mathtt{sup}(a)$ *and* $\beta'' = \mathtt{acc}(a)) \Rightarrow$

$\vdash \|\beta''\| : \{a\}$ (13)

$\cdot; \|\beta''\| \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4$ (14)

By IH, (3), (13), and (14),

$(\beta' = \mathtt{sup}(a)$ *and* $\beta'' = \mathtt{acc}(a)) \Rightarrow a \in A_4$ (15)

By (15) and (12),

$(\beta' = \mathtt{sup}(a)$ *and* $\beta'' = \mathtt{acc}(a)) \Rightarrow a \in A$ (16)

By (6), (9), and (16),

$a \in A$ (17)

■

Lemma 19 (Preservation of Computations 1)
If

1. $\vdash \sigma : B$
2. $\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$
3. $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$

then

1. $\vdash \sigma' : B'$
2. $\cdot; B' \sqcup A_r \vdash^{A_r} E' : \tau, A$

Proof: By induction on the derivation of $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$. We have chosen three representative cases to show below; the others use the same techniques.

Case (E-Ret):
By assumption,
$\vdash \sigma : B$ (1)
$\cdot; B \sqcup A_r \vdash^{A_r} M_1 : \tau, A$ (2)
$M_1 \longmapsto M_2$ (3)
By Lemma 13 and (2),
$\vdash M_1 : \tau$ (4)
By Lemma 9, (3), and (4),
$\vdash M_2 : \tau$ (5)
By (5) and rule (SE-Ret),
$\cdot; B \sqcup A_r \vdash^{A_r} M_2 : \tau, \emptyset$ (6)
By (6) and rule (SE-Sub),
$\cdot; B \sqcup A_r \vdash^{A_r} M_2 : \tau, A$ (7)

Case (E-Let2):
By assumption,
$\vdash \sigma : B$ (1)
$\cdot; B \sqcup A_r \vdash^{A_r} \texttt{let}\ \{x\} = \{v\}^{A'}\ \texttt{in}\ E_1 : \tau, A$ (2)
By Lemma 13 and (2),
$\vdash \{v\}^{A'} : \mathcal{M}^{A'_r}_{A_2}(\tau'')$ (3)
$x : \tau''; \diamond \vdash^{A_r} E_1 : \tau', A_1$ (4)
$\vdash \tau' \leq \tau$ (5)
$A_1 \cup A_2 \subseteq A$ (6)
By Lemma 3 and (3),
$\cdot; \diamond \vdash^{A'_r} v : \tau'_1, A'_e$ (7)
$\vdash \mathcal{M}^{A'_r}_{A'_e}(\tau'_1) \leq \mathcal{M}^{A'_r}_{A_2}(\tau'')$ (8)
By Lemma 2 and (8),
$\vdash \tau'_1 \leq \tau''$ (9)
By Lemma 13 and (7),
$\vdash v : \tau'_1$ (10)
By (9), (10), and rule (S-Sub),
$\vdash v : \tau''$ (11)
By Lemma 1, (4), and (11),
$\cdot; \diamond \vdash^{A_r} E_1[v/x] : \tau', A_1$ (12)
By (5), (6), (12), and rule (SE-Sub),
$\cdot; B \sqcup A_r \vdash^{A_r} E_1[v/x] : \tau, A$ (13)

Case (E-SeqCon3):
By assumption,
$\vdash \sigma : B$ (1)
$\cdot; B \sqcup A_r \vdash^{A_r} (E_1 \triangle^{A_1, A_2} E_2) : \tau, A$ (2)
$E' = E'_1 \triangle^{A_1, A_2} (\beta' \blacktriangleright E_2)$ (3)
$\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma' \triangleright E'_1$ (4)

$\beta' = \texttt{ins}(a)$ *or* $\beta' = \texttt{acc}(a)$ (5)

$a \in A_2$ (6)

By Lemma 13, (2),

$\cdot; B' \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$ (7)

$\cdot; \diamond \vdash^{A_2} E_2 : \tau_2, A_4$ (8)

$A_1 \cup A_2 = A_r$ (9)

$(B', \tau_1 \times \tau_2, A_3 \cup A_4) \ll (B \sqcup A_r, \tau, A)$ (10)

By Lemma 16, (9), and (10),

$B \sqcup A_1 \precsim B' \sqcup A_1$ (11)

By (7), (11), and rule (SE-Sub),

$\cdot; B \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$ (12)

By IH, (1), (12), and (4),

$\vdash \sigma' : B''$ (13)

$\cdot; B'' \sqcup A_1 \vdash^{A_1} E_1' : \tau_1, A_3$ (14)

By (6), (8), and rule (SE-Sub),

$\cdot; \{a\} \vdash^{A_2} E_2 : \tau_2, A_4$ (15)

By Lemma 18, (4), (1), and (12),

$(\beta' = \texttt{ins}(a)) \Rightarrow a \in A_3$ (16)

By (6), (9), (14), (15), (3), and rule (SE-SeqCon3),

$(\beta' = \texttt{acc}(a)) \Rightarrow \cdot; B'' \vdash^{A_r} E' : \tau_1 \times \tau_2, A_3 \cup A_4$ (17)

By (6), (9), (14), (15), (16), (3), and rule (SE-SeqCon2),

$(\beta' = \texttt{ins}(a)) \Rightarrow \cdot; B'' \vdash^{A_r} E' : \tau_1 \times \tau_2, A_3 \cup A_4$ (18)

By (5), (17), and (18),

$\cdot; B'' \vdash^{A_r} E' : \tau_1 \times \tau_2, A_3 \cup A_4$ (19)

By (10), (19), and rule (SE-Sub),

$\cdot; B'' \vdash^{A_r} E' : \tau, A$ (20)

By (20) and definition of valid computation-typing judgments,

$B'' \subseteq A_r$ *or* $B'' = \diamond$ (21)

By (20), (21), and definition of $\sqcup$,

$\cdot; B'' \sqcup A_r \vdash^{A_r} E' : \tau, A$ (22)

Result is from (13) and (22)

■

Lemma 20 (Preservation of Computations 2)
If $\vdash^{A_r} \sigma \triangleright E$ ok *and* $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$ *then* $\vdash^{A_r} \sigma' \triangleright E'$ ok.

Proof: Immediate by Lemmas 17 and 19. ■

Lemma 21 (Progress of Computations 1)
If $\vdash \sigma : B$ *and* $\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$ *then*

1. $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$, *or*
2. $E = v$, *or*
3. $E = \texttt{halt}$

Proof: By induction on the derivation of $\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$. We have chosen four representative cases to show below; the others use the same techniques.

Case (SE-Let1):

By assumption,

$$\vdash \sigma : B \tag{1}$$
$$E = \texttt{let}\ \{x\} = M \texttt{in}\ E_1 \tag{2}$$
$$\vdash M : \mathcal{M}^{A'_r}_{A_2}(\tau') \tag{3}$$

By Lemma 11 and (3),

$$M \longmapsto M' \ or\ M = v \tag{4}$$

By Lemma 3 and (3),

$(M = v) \Rightarrow$

$$M = \{E_2\}^{A'_r} \tag{5}$$
$$\cdot; \diamond \vdash^{A'_r} E_2 : \tau'_1, A'_2 \tag{6}$$

By (6) and rule (SE-Sub),

$$(M = v) \Rightarrow \cdot; B \sqcup A'_r \vdash^{A'_r} E_2 : \tau'_1, A'_2 \tag{7}$$

By IH, (1), and (7),

$$(M = v) \Rightarrow (E_2 = v\ or\ E_2 = \texttt{halt}\ or\ \vdash^{A'_r} \sigma \triangleright E_2 \overset{\beta'}{\longmapsto} \sigma' \triangleright E'_2) \tag{8}$$

By (4), (5), (8), and rules (E-Let1)-(E-Let6),

$$\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \tag{9}$$

Case (SE-Let2):

By assumption,

$$\vdash \sigma : B \tag{1}$$
$$E = \texttt{let}\ \{x\} = (a \triangleright E_1)^{A'} \texttt{in}\ E_2 \tag{2}$$
$$\cdot; \{a\} \vdash^{A'} E_1 : \tau', A_1 \tag{3}$$
$$a \in A' \tag{4}$$

By (3), (4), and definition of $\sqcup$,

$$\cdot; \{a\} \sqcup A' \vdash^{A'} E_1 : \tau', A_1 \tag{5}$$

By rule (Seq-Act),

$$\vdash a : \{a\} \tag{6}$$

By IH, (5), and (6),

$$\vdash^{A'} a \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma'' \triangleright E'_1\ or\ E_1 = v\ or\ E_1 = \texttt{halt} \tag{7}$$

By Lemma 14,

$$(\vdash^{A'} a \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma'' \triangleright E'_1) \Rightarrow (\sigma'' = a\ or\ \sigma'' = \cdot) \tag{8}$$

By (7), (8), and rules (E-Let7)-(E-Let10),

$$\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E' \tag{9}$$

Case (SE-Acc):

By assumption,

$$\vdash \sigma : B \tag{1}$$
$$E = \texttt{ok}; E' \tag{2}$$
$$\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A \tag{3}$$
$$B \sqcup A_r \neq \diamond \tag{4}$$

By (3) and definition of valid computation-typing judgments,

$$B \sqcup A_r \subseteq A_r\ or\ B \sqcup A_r = \diamond \tag{5}$$

By (4) and (5),

$$B \sqcup A_r \subseteq A_r \tag{6}$$

By (6) and definition of $\sqcup$,

$$B \subseteq A_r \tag{7}$$
$$B \neq \diamond \tag{8}$$

By Lemma 15, (1), and (8),

$\sigma = a; \sigma'$ (9)

$B = \{a\}$ (10)

By (7) and (10),

$a \in A_r$ (11)

By (9), (11), and rule (E-Acc),

$\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$ (12)

Case (SE-ParCon):

By assumption,

$\vdash \sigma : B$ (1)

$E = E_1 \wedge^{A_1, A_2} E_2$ (2)

$\cdot; B \sqcup A_r \vdash^{A_r} E : \tau, A$ (3)

$\cdot; (B \sqcup A_r) \sqcup A_1 \vdash^{A_1} E_1 : \tau_1, A_3$ (4)

$\cdot; (B \sqcup A_r) \sqcup A_2 \vdash^{A_2} E_2 : \tau_2, A_4$ (5)

$A_1 \cap A_4 = A_2 \cap A_3 = \emptyset$ (6)

By Lemma 15 and (1),

$(\sigma = a; \sigma' \text{ and } B = \{a\})$ *or* $(\sigma = \cdot \text{ and } B = \diamond)$ (7)

By (3) and definition of valid computation-typing judgments,

$B \sqcup A_r \subseteq A_r$ *or* $B = \diamond$ (8)

By (8) and definition of $\sqcup$,

$B \subseteq A_r$ *or* $B = \diamond$ (9)

By (7), (9), and definition of $\sqcup$,

$(\sigma = a; \sigma' \text{ and } B \sqcup A_r = \{a\})$ *or* $(\sigma = \cdot \text{ and } B \sqcup A_r = \diamond)$ (10)

By (10) and rules (Seq-Empty) and (Seq-Act),

$\vdash \sigma : B \sqcup A_r$ (11)

By IH, (11), and (4),

$\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma_1 \triangleright E_1'$ *or* $E_1 = v$ *or* $E_1 = \texttt{halt}$ (12)

By IH, (11), and (5),

$\vdash^{A_2} \sigma \triangleright E_2 \overset{\beta''}{\longmapsto} \sigma_2 \triangleright E_2'$ *or* $E_2 = v$ *or* $E_2 = \texttt{halt}$ (13)

By Lemma 18, (11), and (4),

$(\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma_1 \triangleright E_1' \text{ and } (\beta' = \texttt{ins}(a) \text{ or } \beta' = \texttt{sup}(a))) \Rightarrow a \in A_3$ (14)

By Lemma 18, (11), and (5),

$(\vdash^{A_2} \sigma \triangleright E_2 \overset{\beta''}{\longmapsto} \sigma_2 \triangleright E_2' \text{ and } (\beta'' = \texttt{ins}(a) \text{ or } \beta'' = \texttt{sup}(a))) \Rightarrow a \in A_4$ (15)

By (6), (14), and definition of $\|\beta'\|$,

$(\vdash^{A_1} \sigma \triangleright E_1 \overset{\beta'}{\longmapsto} \sigma_1 \triangleright E_1' \text{ and } \beta' \neq \texttt{acc}(a)) \Rightarrow \|\beta'\| \notin A_2$ (16)

By (6), (15), and definition of $\|\beta''\|$,

$(\vdash^{A_2} \sigma \triangleright E_2 \overset{\beta''}{\longmapsto} \sigma_2 \triangleright E_2' \text{ and } \beta'' \neq \texttt{acc}(a)) \Rightarrow \|\beta''\| \notin A_1$ (17)

By (12), (13), (16), (17), and rules (E-ParCon1)-(E-ParCon5),

$\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$ (18)

■

Lemma 22 (Progress of Computations 2)
If $\vdash^{A_r} \sigma \triangleright E$ `ok` *then*

1. $\vdash^{A_r} \sigma \triangleright E \overset{\beta}{\longmapsto} \sigma' \triangleright E'$, *or*

2. $E = v$, *or*
3. $E = \texttt{halt}$

Proof: Immediate by Lemmas 17 and 21. ■

Lemma 23 (Preservation: Running System)
If $\vdash \mathcal{S}$ ok *and* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$ *then* $\vdash \mathcal{S}'$ ok.

Proof: By inspection of the operational semantics of running systems, along with inversion of (Run-Ok), Lemmas 17 and 20, and the rules for system typing. ■

Lemma 24 (Progress: Running System)
If $\vdash \mathcal{S}$ ok *then* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$, *or* $\mathcal{S} = v$, *or* $\mathcal{S} = \texttt{halt}$.

Proof: Immediate by inversion of (Run-Ok), Lemma 22, and examination of the operational semantics of running systems. ■

Definition 25 ($\overset{\sigma}{\hookrightarrow}$*)
We extend the single-step dynamic semantics of running systems to a multi-step semantics with the usual reflexive and transitive rules.

$$\frac{}{\mathcal{S} \dot{\hookrightarrow}^* \mathcal{S}} \qquad \text{(R*-Reflex)}$$

$$\frac{\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}' \quad \mathcal{S}' \overset{\sigma'}{\hookrightarrow}^* \mathcal{S}''}{\mathcal{S} \overset{\sigma;\sigma'}{\hookrightarrow}^* \mathcal{S}''} \qquad \text{(R*-Trans)}$$

Definition 26 (Stuck System)
A system $\mathcal{S}$ *is "stuck" if* $\mathcal{S} \neq v$ *and* $\mathcal{S} \neq \texttt{halt}$ *and there does not exist a system* $\mathcal{S}'$ *such that* $\mathcal{S} \overset{\sigma}{\hookrightarrow} \mathcal{S}'$.

Theorem 27 (System Safety)
If $\vdash \mathcal{S}$ ok *and* $\mathcal{S} \overset{\sigma}{\hookrightarrow}^* \mathcal{S}'$ *then* $\vdash \mathcal{S}'$ ok *and* $\mathcal{S}'$ *is not stuck.*

Proof: Immediate by Lemmas 23 and 24. ■