Reproduction of some key results in Leland et al.

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This short monograph describes a reproduction of some of the key results in the famous Leland et al. Bellcore study of Ethernet traffic [1]. The results herein were all generated using the traffic analysis tools available at [3]. The raw data for the Bellcore study [2] is stored in text file format as `<time-stamp, packet size>` pairs. Figure 1 shows the first dozen entries from file `pAugTL.z`.

```
0.001340  1090
0.001508   174
0.004176   162
0.008140   174
0.011036   162
0.015072   174
0.017892   162
0.020604   150
0.022032   174
0.024300    90
0.024752   162
0.027356   150
```

Figure 1 - The raw Bellcore data (first dozen entries in `pAugTL.z`)

The time stamps of the Bellcore raw data are cumulative. This raw data was stripped of the packet size information and then converted to packet counts per 0.01 seconds. The tool used to convert cumulative time stamps to packet counts was `ttoc1.c`. The converted packet count file was named `bell.dat` and contained 314283 count values. Figure 2 shows the batch file used to compute the autocorrelation for `bell.dat` and the mean, variance, and R/S values for `bell.dat` for aggregations of 10x, 100x, and 1000x. The programs `autoc.c`, `block.c`, `summary1.c`, and `rs.c` are all available at [3]. The program `rs.c` computes a single *R*/*S* value for a series *X*. For `block.c` the value of *M* was set to 10. The analysis of Figure 2 on `bell.dat` was repeated on a file `poisson.dat` where `poisson.dat` contained 314283 Poisson distributed values of mean 3.184662 (3.181846 is the mean value of `bell.dat`). The values in `poisson.dat` were generated using `genexp.c` and `ttoc2.c` [3]. The execution time for the batch file of Figure 2 was about 10 seconds on a 866-Mhz PentiumIII Dell PC. The raw `result.txt` files for the analysis of `bell.dat` and `poisson.dat` are in Appendix A and B.

```
autoc    < bell.dat   > result.txt
block    < bell.dat   > b10.dat
block    < b10.dat    > b100.dat
block    < b100.dat   > b1000.dat
summary1 < bell.dat  >> result.txt
summary1 < b10.dat   >> result.txt
summary1 < b100.dat  >> result.txt
summary1 < b1000.dat >> result.txt
rs       < bell.dat  >> result.txt
rs       < b10.dat   >> result.txt
rs       < b100.dat  >> result.txt
rs       < b1000.dat >> result.txt
```

Figure 2 - Batch file for analysis of `bell.dat`

The Hurst parameter (H) is estimated from the slope of the line of a plot of Log(N) versus Log(R/S). For `bell.dat` N = 314283 values, for `b10.dat` N = 31428, for `b100.dat` N = 3142, and for `b1000.dat` N = 314. Table 1 shows the mean and variance for the Bellcore data and the Poisson distributed values. Figure 3 shows the autocorrelation for `bell.dat` and `poisson.dat` for the first 1000 lags. Finally, Figure 4 shows the plotted Log(M) versus Log(R/S) for the Bellcore data and the Poisson distributed values. Shown on the graphs is the equation of the fitted line with the R^2 fit value. We note the estimated H values from the slope of the fitted line, H = 0.52 for Poisson and H = 0.83 from the Bellcore data. In [1] an H value of 0.85 is reported for the Bellcore data and H = 0.50 is expected for Poisson values.

Table 1 - Mean and variance for Bellcore data and Poisson distributed values

	Bellcore data		Poisson values	
Blocking	Mean	Variance	Mean	Variance
1	3.182	6.803	3.182	3.172
10	3.182	3.235	3.182	0.319
100	3.182	1.311	3.182	0.032
1000	3.182	0.638	3.182	0.003

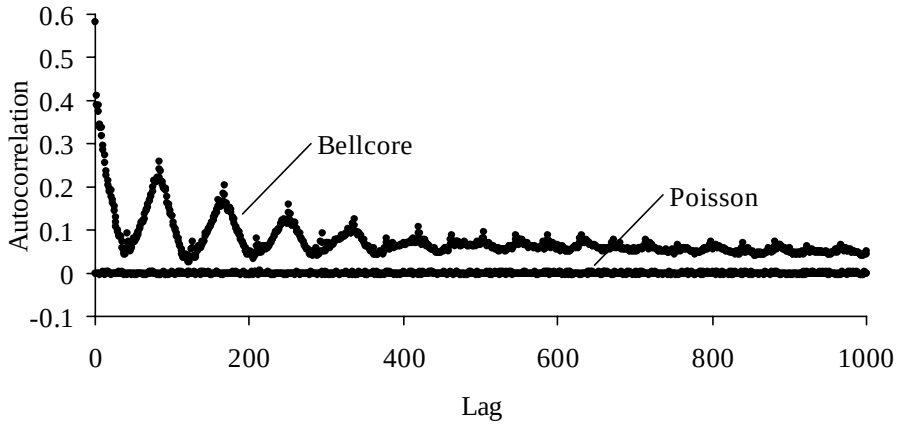

Figure 3 - Autocorrelation for the Bellcore data and Poisson distributed values

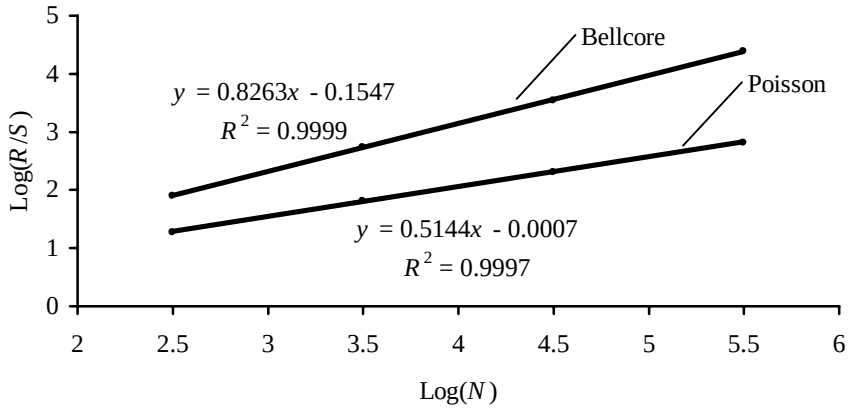

Figure 4 - Hurst parameter estimate for the Bellcore data and Poisson distributed values

References

[1] W. Leland, M. Taqqu, W. Willinger, and D. Wilson, "On the Self-Similar Nature of Ethernet Traffic (Extended Version)," *IEEE/ACM Transactions on Networking*, Vol. 2, No. 1, pp. 1 - 15, February 1994.

[2] BC - 4 million-packet traces of LAN and WAN traffic seen on an Ethernet. URL: *http://ita.ee.lbl.gov/html/contrib/BC.html*.

[3] Tools page for Kenneth J. Christensen, 1998. URL: *http://www.csee.usf.edu/~christen/toolpage.html*.

Appendix A - Raw data for Bellcore data analysis

```
------------------------------------------------ autoc.c -----
  Autocorrelation for lag    1 = 0.582134
  Autocorrelation for lag    2 = 0.412211
  Autocorrelation for lag    3 = 0.388460
  Autocorrelation for lag    4 = 0.388352
  Autocorrelation for lag    5 = 0.373063
  Autocorrelation for lag    6 = 0.345029
  Autocorrelation for lag    7 = 0.336514
  Autocorrelation for lag    8 = 0.338726

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  Autocorrelation for lag  999 = 0.045363
  Autocorrelation for lag 1000 = 0.050316
-----------------------------------------------------------------
------------------------------------------------ summary1.c -----
  Total of 314283 values
    Minimum  = 0.000000 (position = 314222)
    Maximum  = 23.000000 (position = 304297)
    Sum      = 1000000.000000
    Mean     = 3.181846
    Variance = 6.802940
    Std Dev  = 2.608245
    CoV      = 0.819727
-----------------------------------------------------------------
------------------------------------------------ summary1.c -----
  Total of 31428 values
    Minimum  = 0.000000 (position = 29730)
    Maximum  = 15.700000 (position = 30429)
    Sum      = 99999.400000
    Mean     = 3.181857
    Variance = 3.235453
    Std Dev  = 1.798736
    CoV      = 0.565310
-----------------------------------------------------------------
------------------------------------------------ summary1.c -----
  Total of 3142 values
    Minimum  = 0.470000 (position = 1383)
    Maximum  = 8.380000 (position = 808)
    Sum      = 9996.920000
    Mean     = 3.181706
    Variance = 1.310800
    Std Dev  = 1.144902
    CoV      = 0.359839
-----------------------------------------------------------------
------------------------------------------------ summary1.c -----
  Total of 314 values
    Minimum  = 1.170000 (position = 247)
    Maximum  = 5.923000 (position = 81)
    Sum      = 999.081000
    Mean     = 3.181787
    Variance = 0.638428
    Std Dev  = 0.799017
    CoV      = 0.251122
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 24566.823936 for series X of 314283 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 3562.161561 for series X of 31428 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 559.144809 for series X of 3142 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 80.093527 for series X of 314 values
-----------------------------------------------------------------
```

Appendix B - Raw data for Poisson values analysis

```
------------------------------------------------ autoc.c -----
  Autocorrelation for lag    1 = 0.000188
  Autocorrelation for lag    2 = 0.000509
  Autocorrelation for lag    3 = 0.000899
  Autocorrelation for lag    4 = 0.000749
  Autocorrelation for lag    5 = 0.000969
  Autocorrelation for lag    6 = -0.001453
  Autocorrelation for lag    7 = -0.002162
  Autocorrelation for lag    8 = 0.001732

<SNIP SNIP>

  Autocorrelation for lag  999 = -0.001198
  Autocorrelation for lag 1000 = -0.000421
-----------------------------------------------------------------
------------------------------------------- summary1.c -----
  Total of 314283 values
    Minimum  = 0.000000 (position = 314269)
    Maximum  = 14.000000 (position = 275106)
    Sum      = 1000087.000000
    Mean     = 3.182122
    Variance = 3.171790
    Std Dev  = 1.780952
    CoV      = 0.559674
-----------------------------------------------------------------
------------------------------------------- summary1.c -----
  Total of 31428 values
    Minimum  = 1.200000 (position = 7664)
    Maximum  = 5.500000 (position = 16423)
    Sum      = 100007.800000
    Mean     = 3.182124
    Variance = 0.318543
    Std Dev  = 0.564396
    CoV      = 0.177364
-----------------------------------------------------------------
------------------------------------------- summary1.c -----
  Total of 3142 values
    Minimum  = 2.570000 (position = 1691)
    Maximum  = 3.810000 (position = 771)
    Sum      = 9998.290000
    Mean     = 3.182142
    Variance = 0.032006
    Std Dev  = 0.178901
    CoV      = 0.056220
-----------------------------------------------------------------
------------------------------------------- summary1.c -----
  Total of 314 values
    Minimum  = 2.997000 (position = 165)
    Maximum  = 3.370000 (position = 298)
    Sum      = 999.184000
    Mean     = 3.182115
    Variance = 0.003220
    Std Dev  = 0.056743
    CoV      = 0.017832
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 659.863899 for series X of 314283 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 207.271090 for series X of 31428 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 65.151941 for series X of 3142 values
-----------------------------------------------------------------
------------------------------------------------- rs.c -----
  R/S = 18.708043 for series X of 314 values
-----------------------------------------------------------------
```