**Programming Languages (COP 4020/CIS 6930) [Fall 2014]**

Assignment III

**Objectives**
1. To gain experience writing inference rules in deductive systems.
2. To practice proving properties of judgments by induction on their derivations.

**Due Date:** Tuesday, September 23, 2014 (at the beginning of class, 5:00pm).

**Assignment Description**
Do the following by yourself.
(1) Define a deductive system having two judgment forms:
a) Judgments of the form *N Nat* are valid iff N is a natural number. Just use the same 2 inference rules we discussed in class.
b) Judgments of the form $N_1 - N_2 = N_3$ are valid iff subtracting natural-number $N_2$ from natural-number $N_1$ produces natural-number $N_3$. For example, S(S(S(Z)))-S(S(Z))=S(Z) should be derivable, but for all N, S(S(Z))-S(S(S(Z)))=N should not be derivable. Your rules for subtraction judgments can implicitly assume that all numbers involved are natural numbers; your rules for subtraction judgments therefore don't have to contain judgments of the form *N nat*. (Essentially, we're assuming that, in subtraction judgments, the symbol N always refers to a valid natural number.)

(2) Using your definitions from Step (1), formally prove the following Lemma A.

Lemma A. $\forall$ N: (N nat $\Rightarrow$ N-N=Z)

(3) [This step is for graduate students; undergrads may complete this step for +10% extra credit] Again using your definitions from Step (1), formally prove the following Lemma B.

Lemma B. $\forall N_1, N_2, N_3$: ($N_1$-S($N_2$)=$N_3$ $\Rightarrow$ $N_1$-$N_2$=S($N_3$))

(4) Using your definitions from Step (1), formally prove the following Theorem C.

Theorem C. $\forall N_1, N_2, N_3$: ($N_1$-$N_2$=$N_3$ $\Rightarrow$ $N_1$-$N_3$=$N_2$)

If helpful, your proof of Theorem C can assume that Lemmas A and B hold.

**Grading Notes**
Partial credit is always possible. If you get stuck, just explain informally whatever ideas you're having trouble stating formally.

**Submission Notes**
- Turn in a hardcopy (handwritten or printed) version of your solutions. Please do not email solutions or upload them into Canvas.
- Write the following pledge at the end of your submission: "I pledge my Honor that I have not cheated, and will not cheat, on this assignment." Sign your name after the pledge. Not including this pledge will lower your grade 50%.
- You may submit solutions up to 2 days late (i.e., by 5pm on Thursday, September 25) with a 15% penalty.
- If you think there's a chance you'll be absent or late for class on the date this assignment is due, you're welcome to submit solutions early by giving them to me or a TA before or after class, or during any of our office hours.