

Programming Languages [Fall 2014] Test II

NAME: _____

Instructions:

- 1) This test is 6 pages in length.
- 2) You have 75 minutes to complete and turn in this test.
- 3) Short-answer questions include a guideline for how many sentences to write. Respond in complete English sentences.
- 4) This test is closed books, notes, papers, friends, neighbors, etc.
- 5) Use the backs of pages in this test packet for scratch work. If you write more than a final answer in the area next to a question, circle your final answer.
- 6) Write and sign the following: "I pledge my Honor that I have not cheated, and will not cheat, on this test."

Signed: _____

1. [5 points]

What does it mean for a PL to be weakly typed? [1-2 sentences]

2. [75 points]

Let L be a call-by-name, simply typed lambda calculus with base type *int*.

The syntax for expressions in L is: $e ::= e e' \mid x \mid n \mid \lambda x:\tau.e$

(a) Using the following SML definitions for L:

```
datatype typ = Int | Arr of typ*typ;
datatype exp = App of exp*exp | Var of string | Int of int
             | Lam of string*typ*exp;
exception captured;
```

implement an SML function `sub:exp->string->exp->exp` such that `sub e x e'` returns $[e/x]e'$ unless a variable gets captured, in which case *captured* gets raised.

(b) Define L's static semantics.

(c) Define L's SOS-style dynamic semantics ($e \rightarrow e'$).

(d) Define L's big-step operational semantics ($e \Downarrow v$).

(e) Formally state all the standard type-safety lemmas/theorems/corollaries for L.

(f) Using your rules for L, as well as the following:

$$\boxed{e \rightarrow^* e'}$$

$$\frac{}{e \rightarrow^* e} \text{ Re}$$

$$\frac{e \rightarrow e' \quad e' \rightarrow^* e''}{e \rightarrow^* e''} \text{ Tr}$$

prove that $\forall e, v: (e \rightarrow^* v) \Rightarrow (e \Downarrow v)$.

3. [20 points]

a) Encode a ternary logic (three-valued logic) into the untyped lambda calculus. The three values are: T (true), F (false), and B (both). Besides for the values themselves, provide encodings for $\text{AND}(v1,v2)$ and $\text{OR}(v1,v2)$, defined below. Both operations must be short circuit (lazy).

v1	v2	AND(v1,v2)	OR(v1,v2)
T	T	T	T
T	F	B	T
T	B	B	T
F	T	B	T
F	F	F	F
F	B	B	B
B	T	B	T
B	F	B	B
B	B	B	B

b) Using the call-by-value strategy and your response to Part (a), trace the evaluation of $\text{AND}(B,\text{OR}(F,T))$ to B. Show each step of the evaluation and underline redexes.

[Undergraduates stop here. The following problem is for graduate students.]

4. [12 points]

Returning to the language L of Problem 2, assume the following lemmas hold.

Lemma 1. $\forall e_1, e_2, e_3: (e_1 \rightarrow^* e_2 \wedge e_2 \rightarrow^* e_3) \Rightarrow (e_1 \rightarrow^* e_3)$

Lemma 2. $\forall e_1, e_1', e_2: (e_1 \rightarrow^* e_1') \Rightarrow (e_1 e_2 \rightarrow^* e_1' e_2)$

Now prove that $\forall e, v: (e \Downarrow v) \Rightarrow (e \rightarrow^* v)$.