

# Programming Languages [Fall 2018]

## Test II

NAME: \_\_\_\_\_

### Instructions:

- 1) This test is 7 pages in length.
- 2) You have 75 minutes to complete and turn in this test.
- 3) Short-answer and essay questions include a guideline for how much to write. Respond in complete English sentences and avoid using bulleted and itemized lists.
- 4) For full credit on ML-response questions, implementations must avoid the @ operator and run in linear ( $O(n)$ ) time.
- 5) This test is closed books, notes, laptops, phones, smartwatches, friends, neighbors, etc.
- 6) Use the backs of pages in this test packet for scratch work. If you write more than a final answer in the area next to a question, circle your final answer.
- 7) Write and sign the following: "I pledge my Honor that I have not cheated, and will not cheat, on this test."

\_\_\_\_\_  
\_\_\_\_\_

Signed: \_\_\_\_\_

1. [7 points]

What is a programming language? How is one defined? [2-3 sentences]

2. [18 points] [Essay]

$\forall P \in \{\lambda_{ST}, \lambda_{UT}, \text{diML}, \text{STERLING}, L\}$ : explain (at the level discussed in class) why P is or is not Turing complete. L is the PL from the theory assignments.

3. [5 points]

State the standard substitution lemma and technique for proving it.

4. [20 points]

a) Encode a right-associative short-circuit NOR operator for Church booleans. Short-circuit means that the operator only evaluates operands when necessary (like `&&` in C). *Don't* use abbreviations; e.g., don't write "true" as an abbreviation for a  $\lambda_{UT}$  expression.  
 $e_1 \text{ NOR } e_2 \equiv$

b) Now trace evaluation of  $(\text{false NOR false}) \text{ NOR } (\text{false NOR false})$  using left-to-right, normal-order evaluation. Underline redexes and *do* use abbreviations when convenient.

5. [20 points]

For this problem, N always refers to a natural number, so you never need to write N nat.

a) Define inference rules for deriving sum and inequality relationships between natural numbers. The judgment forms are  $N_1+N_2=N_3$  and  $N_1\neq N_2$ .

b) Prove that different addends produce different sums.

**Theorem.**  $\forall N_1, N_2, N_3, N_4, N_5: ((N_1+N_2=N_3 \wedge N_1+N_4=N_5 \wedge N_2\neq N_4) \Rightarrow N_3\neq N_5)$

6. [20 points]

Define higher-order abstract syntax for  $\lambda_{ST}$  having base type `nat`.

7. [10 points]

Recall that `porder L` will “reorder `L`, placing it in ‘pong’ order, that is, bouncing between left and right elements. For example, `porder [1,2,3,4,5]` returns `[1,5,2,4,3]` and `porder [1.1,2.2]` returns `[1.1,2.2]`.” Let’s generalize `porder` to `porder'`, which takes 3 (curried) arguments: a list `L`, a boolean flag `b` indicating whether to start on the left or right side, and an integer `s` indicating how many elements to skip at a time. For example,

- `porder' [1,2,3,4,5] true 1` returns `[1,5,2,4,3]`
- `porder' [1,2,3,4,5] true 2` returns `[1,5,3]`
- `porder' [1,2,3,4,5] false 1` returns `[5,1,4,2,3]`
- `porder' [1,2,3,4,5,6,7,8] false 2` returns `[8,1,6,3]`
- `porder' L b s`, where  $s < 1$ , returns `nil`

Implement `porder'` subject to the constraints of Assignment I (e.g., no library-fun. calls).

**[Undergraduates stop here. The remaining problem is for graduate students.]**

8. [12 points]

State, and prove any 3 cases of, the Weakening Lemma for  $\lambda_{ST}$  having base type  $\text{nat}$ .