

4.1-1:

$$T(n) = T(\lceil n/2 \rceil) + 1$$

$$T(n/2) = T(\lceil n/4 \rceil) + 1$$

$$\begin{aligned}\therefore T(n) &= T(\lceil n/4 \rceil) + 1 + 1 \\ &= T(\lceil n/4 \rceil) + 2\end{aligned}$$

Similarly,

$$T(n) = T(\lceil n/8 \rceil) + 3$$

$$\text{or } T(n) = T(\lceil n/2^m \rceil) + m$$

$$\text{lets say, } 2^m = n$$

$$\therefore m = \lg n$$

$$\therefore T(n) = T(\lceil n/n \rceil) + \lg n$$

$$T(n) = O(\lg n) \quad \text{proved}$$