4.1-2: We saw that solution of $T(n) = 2T(n/2) + n$ is $O(n\lg n)$. Show that the solution of this recurrence is also $\Omega(n\lg n)$. Conclude that the solution is $\Theta(n\lg n)$

$$T(n) = 2.T(n/2) + n$$

$$\therefore T(n/2) = 2.T(n/4) + n/2$$

$$\therefore T(n) = 2.\{2.T(n/4) + n/2\} + n$$

or, $T(n) = 4.T(n/4) + 2n.$

or, $T(n) = 8.T(n/8) + 3n$

or $T(n) = 2^m.T(n/2^m) + m.n$

let us say $2^m = n$ $\therefore$ $m = \lg n$

$$T(n) = 2^{\lg n}\, T(n/n) + n.\lg n$$

$$T(n) = n.\lg n + n$$

$\therefore$ if for some $n (= n_0)$, there exists c,

$c.n\lg n < n.\lg n + n$ is true

$$\therefore T(n) = \Omega(n\lg n)$$

& as we know,

if $f(n) = O(g(n))$

& $f(n) = \Omega(g(n))$

then $f(n) = \Theta(g(n))$ proved