Change Point Estimation of Bar Code Signals

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Abstract

Existing methods for bar code signal reconstruction is based on either the local approach or the regularization approach with total variation penalty. We formulate the problem explicitly in terms of change points of the 0-1 step function. The bar code is then reconstructed by solving the nonlinear least squares problem subject to linear inequality constraints, with starting values provided by the local extremas of the derivative of the convolved signal from discrete noisy data. Simulation results show a considerable improvement of the quality of the bar code signal using the proposed hybrid approach over the local approach.

EDICS Category:IMD-ANAL, SAS-SYST

I. Introduction

The ubiquitous alternating black and white strips – the bar code – are now widely used in every day life and industrial process. The problem of recovering a bar code signal $f(t)$ from the noisy signal $y(t)$ detected by a bar code scanner [2] [7] is to construct a one-dimensional $0-1$ step function $f(t)$ given the samples $y_i = y(t_i), i = 1, \ldots, M,$ of the continuous-time observation

$$y(t) = \alpha \cdot G \star f(t) + \epsilon(t), \quad t \in T = [0, 1]$$

where $\alpha > 0$ is the unknown amplitude, the $\epsilon(t)$ is the additive unobservable noise process and

$$G \star f(t) = \int_T G(t-x) f(x) dx,$$

and $G(t)$ is a Gaussian kernel of the convolution:

$$G(t) = \frac{1}{\sigma_0 \sqrt{2\pi}} exp(-\frac{t^2}{2\sigma_0^2}),$$

where $\sigma_0 > 0$ is the unknown standard deviation which increases as the scanner moves away from the bar code. Figure 1 illustrates the results of the convolution with additive noise for a UPC bar code [9] encoding 0123456789.

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This problem differs slightly from standard restoration problems of image processing in that the convolution kernel contains unknown quantities. Thus it is somewhat closer to the blind deconvolution problems.

II. Previous work and our approach

Previous work on the bar code reconstruction problem [9] is based on (a) local approach: finding local minima and maxima in the derivative of

$$s(t) = \alpha \cdot G \star f(t),$$

(b) global approach: regularization methods for ill-posed inverse problems such as total variation based restoration [2].

The approach (a) try to relate the local minima and maxima in $s'(t)$ to the edges of bars which are the change points in $f(t)$. Locating these local extremas is sensitive to noise $\epsilon(t)$. Furthermore, these local extrema are difficult to relate to the true change points of $f(t)$ due to 'convolution distortion' [7]. Such techniques use only local information and would have difficulty to detect thin bars closely adjacent to each other. For example, in Figure 1, the $s(t)$ is near flat around location 550 even though three adjacent thin bars stand there.

To overcome these shortcomings, approach (b) in [2] try to recover $f(t)$ by regularization using the total variation penalty, a technique commonly used in image restoration literature. It models systematically the interaction of neighboring bars in $f(t)$ under convolution with $G(t)$, as well as the estimation of α and σ_0 from global information contained in the $y(t)$. It is proved in [2] that under certain regularity conditions, the infimum of the total-variation energy is attained. Numerical results show that bar codes from highly degraded signals can be recovered.

The regularization approach in inverse problems must deal with the choice of the regularization parameter, a difficult problem itself. In [2], there are two regularization parameters which need to be chosen. In the numerical results of [2], the regularization parameters are preselected and kept fixed.

We feel all the existing works did not fully utilize the information about $f(t)$: a 0-1 step function. To recover $f(t)$ is to recover the change points of $f(t)$ for $t \in T$. The number of change points in $f(t)$ is twice the number of bars in the bar code. Recovering the $f(t),\ t \in T$ is usually an ill-posed problem, while recovering the change points is a well-posed problem if the number of observations exceed the number of unknown parameters. We did not find the formulation of the bar code deconvolution in terms of the change points explicitly in the existing literature. Therefor we propose a nonlinear least squares solution to the change points of $f(t)$, σ_0 and α with the constraints of the ordered change points. The local approach is used to provide the starting values for the global minimization problem. Our method is a hybrid of local and global approach in spirit.

III. CHANGE POINT ESTIMATION

Assuming the total number of bars of $f(t)$ is the known integer K. For example, $K = 22$ for the UPC bar code in our test problem. Denoting ξ_{2j-1} and ξ_{2j} as the beginning and ending location of the jth bar for $j = 1, \ldots, K$ of the bar code function $f(t)$.

Then $f(t)$ can be defined explicitly as:

$$f(t) = I(\xi_{2j-1} < t \leq \xi_{2j}),\ t \in T,\ j = 1, \ldots, K.$$

where $I()$ is the usual indicator function and

$$0 < \xi_1 < \xi_2 < \ldots < \xi_{2K-1} < \xi_{2K} < 1$$

are the ordered change points.

The goal of bar code reconstruction is to recover the change points $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_{2K-1}, \xi_{2K})^T$ from the observed data $\boldsymbol{y} = (y_1, \ldots, y_M)^T$ at $\boldsymbol{t} = (t_1, \ldots, t_M)^T$, without any knowledge of α and σ_0.

With the special structure of $f(t)$, the convolution $G \star f(t)$ can be explicitly expressed as a function of $\boldsymbol{\xi}$ and σ_0. Thus we have

$$s(t) = \alpha \int_T G(t-x) f(x) dx = \alpha \sum_{j=1}^{K} \int_{\xi_{2j-1}}^{\xi_{2j}} G(t-x) dx.$$

Let

$$r_i = s_i - y_i = \alpha \sum_{j=1}^{K} \int_{\xi_{2j-1}}^{\xi_{2j}} G(t_i - x) dx - y_i$$

be the ith residual, $\boldsymbol{r} = (r_1, \ldots, r_M)^T$ the residual vector and

$$h(\boldsymbol{\xi}, \alpha, \sigma_0) = \frac{1}{2} \sum_{i=1}^{M} r_i^2 = \frac{1}{2} r^T r$$

the residual sums of squares. We seek the least squares solution of $\boldsymbol{\xi}, \sigma_0$ and α. That is to find $\hat{\boldsymbol{\xi}}, \hat{\sigma}_0$ and $\hat{\alpha}$ which minimizes the merit function $h(\boldsymbol{\xi}, \sigma_0, \alpha)$ subject to the required conditions.

More explicitly, the constrained nonlinear least squares problem is

$$min_{\boldsymbol{\xi}, \sigma_0, \alpha} h(\boldsymbol{\xi}, \sigma_0, \alpha) \tag{1}$$

such that

$$0 < \xi_1 < \xi_2 < \ldots < \xi_{2K-1} < \xi_{2K} < 1, \quad \sigma_0 > 0, \quad \alpha > 0.$$

These constraints are simply linear inequality constraints $A[\boldsymbol{\xi}^T, \sigma_0, \alpha]^T < u$ with a sparse matrix A whose elements are

$$\begin{aligned} A[1,1] &= -1, \\ A[i,i-1] &= 1,\ A[i,i] = 1 \text{ for } i = 2, \ldots, 2K, \\ A[2K+1, 2K] &= 1, \\ A(2K+2, 2K+1) &= -1, \\ A(2K+3, 2K+2) &= -1; \end{aligned}$$

and $u = (0, \ldots, 0, 1, 0, 0)^T$ is a $(2K+3)$ column vector.

The recast of the bar code reconstruction into a constrained nonlinear least squares problem enables us to utilize the existing techniques for solving nonlinear least square problem subject to linear inequality constraints in the statistical and numerical analysis literature.

The Fletcher-Xu hybrid Gauss-Newton and BFGS method [4] for nonlinear least squares problem are super linearly convergent. This method along with other five methods for constrained nonlinear least squares problems is implemented in the $clsSolve$ solver of the TOMLAB 4.7 optimization environment [8].

The Gauss-Newton method needs the gradient of the merit function $h(\boldsymbol{\xi}, \sigma_0, \alpha)$, which is the product of the Jacobian matrix of $\boldsymbol{r}$ and $\boldsymbol{r}$. The Jacobian matrix of $\boldsymbol{r}$ is easily obtained by :

$$\begin{aligned} \frac{\partial r_i}{\partial \xi_{2j-1}} &= -\alpha \cdot G(t_i - \xi_{2j-1}), \\ \frac{\partial r_i}{\partial \xi_{2j}} &= \alpha \cdot G(t_i - \xi_{2j}), \\ \frac{\partial r_i}{\partial \sigma_0} &= \frac{\alpha}{\sigma_0} \sum_{j=1}^{K} \int_{\xi_{2j-1}}^{\xi_{2j}} [-1 + \frac{(t_i - x)^2}{\sigma_0^2}] G(t_i - x) dx, \\ \frac{\partial r_i}{\partial \alpha} &= \sum_{j=1}^{K} \int_{\xi_{2j-1}}^{\xi_{2j}} G(t_i - x) dx. \end{aligned}$$

The success of the $(2K+2)$ dimensional global minimization problem (1) heavily depends on good starting values. Our numerical experiments indicated that simple starting values such as equally spaced grids on T for $\boldsymbol{\xi}$ did not give satisfactory solutions. Next we discuss the initial parameter estimation based on the local approach.

IV. Initial Estimation

The local extremas of the derivative of $s(t)$ are close to $\boldsymbol{\xi}$, the edges of the bars. Then the initial estimation of $\boldsymbol{\xi}$ in terms of the local extremas of the derivative signal is the following problem: given the noisy discrete observations of $s(t)$:

$$y_i = s(t_i) + \epsilon_i, \quad i = 1, \ldots, M,$$

finding the local extremas of $s'(t)$.

There are approaches of estimating $s'(t)$ based on different smoothing or denosing methods. Many of these try first to find $\hat{s}(t)$, the estimate of $s(t)$; then using $\hat{s}'(t)$ to estimate $s'(t)$. See, for example, spline regression based method in [10], wavelet denosing based method in [1]. For equally spaced $\{t_i\}$ and when M is a power of 2, there exists a fast algorithm with complexity $O(M)$ to carry out discrete wavelet transform (DWT). In our simulation, we use wavelet thresholding method to estimate $s(t_i)$ first, then estimate $s'(t_i)$ based on $\hat{s}(t_i)$ using a first derivative filter.

After obtaining the initial estimate $\boldsymbol{\xi}_{initial}$ of $\boldsymbol{\xi}$ by the K pairs of local maxima and minima of $\hat{s}'(t_i)$, the initial σ_0, $\sigma_{0initial}$, is estimated by techniques suggested in [5], [6] and [2]. Proposition 1 of [5] suggests approximating σ_0 by the distance from the last local maxima of $\hat{s}(t_i)$ to the last local minima of $\hat{s}'(t_i)$. Proposition 2 of [6] suggests approximating σ_0 by $|\hat{s}'(t^*)/\hat{s}^{(3)}(t^*)|$ where t^* is a point such that $|\hat{s}'(t^*)| \geq |\hat{s}'(t_i)|$ for $i = 1, \ldots, M$. The smaller vale of $\hat{\sigma}_0$ based on the two propositions is used first. If it is outside the reasonable range $[0.001, 0.02]$, then the value 0.0079 is used as suggested in [2] for the true σ_0 ranging from 0.012 to 0.014 which is about the range that the bar code is not blurred too much and is still being able to be recovered.

The initial value of α, $\alpha_{initial}$, is simply the ordinary least squares estimate given the $\boldsymbol{\xi}_{initial}$ and $\sigma_{0initial}$.

V. Simulation results

In the experiment, a 'clean' bar code $f(t)$ was blurred by convolving it with $G(t)$ of known σ_0, amplified by the amplitude $\alpha = 1$, sampled at $t_i = i/M$, for $i = 1, \ldots, M$, followed by the addition of white Gaussian noise $\epsilon_i \sim N(0, \sigma^2)$. The amount of the added noise makes the signal-to-noise ratio $SNR = 20log_{10}(std(\boldsymbol{s})/\sigma)$ at the specified level.

Estimation of $s(t_i)$ is carried out by the soft Wavelet thresholding technique implemented in the Wavelet Toolbox in MATLAB. The thresholds are chosen by a heuristic variant of the Stein's Unbiased Risk Estimate with multiplicative threshold rescaling using a single estimation of level noise based on the finest level wavelet coefficients. The wavelet filter used is db6: the Daubechies wavelet with 6 vanishing moments.

The first derivative filter for estimating $s'(t_i)$ from $\hat{s}(t_i)$ is

$$d1 = [-0.015964, -0.121482, -0.193357,$$
$$0.00, 0.193357, 0.121482, 0.015964];$$

as recently suggested in [3].

Table I shows the Monte-Carlo approximations to $MSE = E(||\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}||_2^2/(2K))$ of our method based on 100 independent simulations as the σ_0 is varied at $0.012, 0.013$ and 0.0133, the sample size M is varied dyadically from $M = 256$ through 1024, and SNR is varied from high to moderate levels.

TABLE I

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}||_2^2/(2K))$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	4.064e-09	8.141e-05	2.423e-04
0.0120	512	1.909e-09	1.036e-06	3.010e-04
0.0120	1024	9.762e-10	9.923e-09	2.558e-04
0.0130	256	1.359e-04	2.291e-04	6.485e-04
0.0130	512	6.773e-05	1.405e-04	8.371e-04
0.0130	1024	5.585e-05	2.189e-04	6.406e-04
0.0133	256	8.152e-05	2.234e-04	9.334e-04
0.0133	512	1.825e-04	2.882e-04	9.294e-04
0.0133	1024	1.130e-04	4.029e-04	7.158e-04

Table II shows the Monte-Carlo approximations to $MSE = E(||\boldsymbol{\xi}_{initial} - \boldsymbol{\xi}||_2^2/(2K))$. The results show a considerable reduction of MSE for $\hat{\boldsymbol{\xi}}$ over $\boldsymbol{\xi}_{initial}$ in some cases. The most significant improvement occured for the case $\sigma_0 = 0.012$ with high SNR.

Figures 1 and 2 present results from two of these experiments. Figure 1 is an example of completely successful reconstruction while the Figure 2 an example that the estimation fails when the noise level or the blur factor σ_0 gets too high.

Table III and IV display the Monte-Carlo approximations to $MSE = E(||\hat{\sigma}_0 - \sigma_0||_2^2)$ and $E(||\sigma_{0initial} - \sigma_0||_2^2)$ respectively.

Table V shows the Monte-Carlo approximations to $MSE = E(||\hat{\alpha} - \alpha||_2^2)$.

TABLE II

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\boldsymbol{\xi}_{initial} - \boldsymbol{\xi}||_2^2/(2K))$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	2.064e-05	1.042e-04	2.947e-04
0.0120	512	1.280e-05	1.729e-05	3.565e-04
0.0120	1024	1.134e-05	1.302e-05	3.183e-04
0.0130	256	2.303e-04	3.242e-04	7.221e-04
0.0130	512	9.190e-05	1.833e-04	8.862e-04
0.0130	1024	1.010e-04	2.803e-04	7.402e-04
0.0133	256	2.122e-04	3.358e-04	1.029e-03
0.0133	512	2.140e-04	3.707e-04	8.854e-04
0.0133	1024	1.899e-04	4.959e-04	8.078e-04

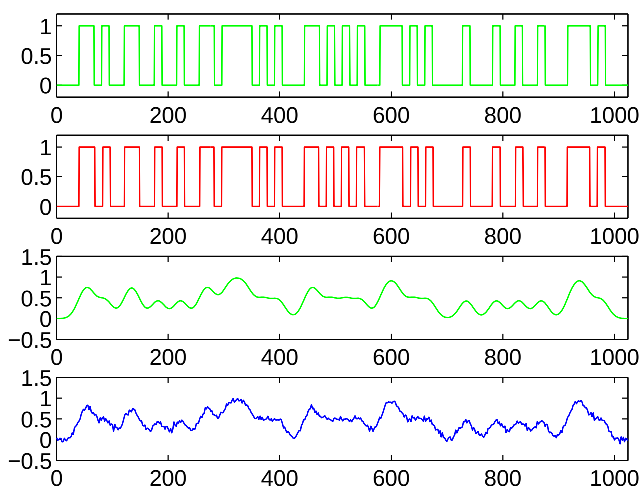

Fig. 1. Top to bottom: the bar code, the reconstructed bar code, the corresponding clean convolved signal, the noisy convolved signal. The true parameter used to generate the corrupted signal: $M = 512$, $\sigma_0 = 0.0118$, $\alpha = 1$, $SNR = 16$. The estimated parameter: $\hat{\sigma}_0 = 0.0115$, $\hat{\alpha} = 0.9962$. The square error $||\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}||_2^2/44 = 8.3e-7$. CPU time: 67 seconds.

Table VI shows the Monte-Carlo approximations to $MSE = E(||\alpha_{initial} - \alpha||_2^2)$.

Note that $\hat{\sigma}_0$ gives much better solution than the initial estimate $\sigma_{0initial}$ in terms of the reduction of MSE. The same is true for $\hat{\alpha}$.

The Computational time for finding the solution is relative fast. For example, for the most time consuming scenario $\sigma_0 = 0.133,\ M = 1024,\ SNR = 21$, the average CPU time is 80 seconds. This is

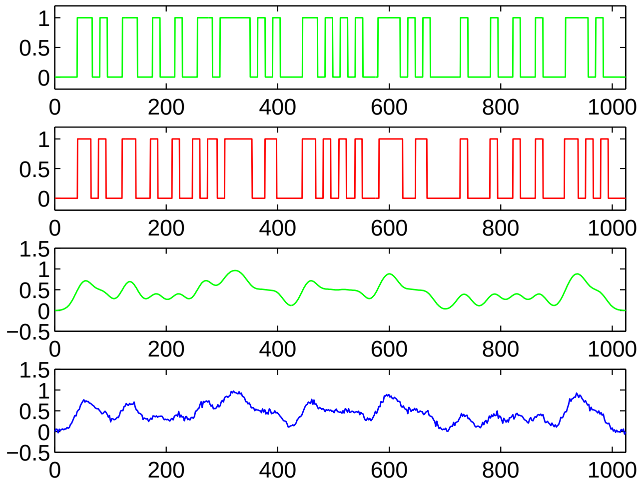

Fig. 2. Top to bottom: the bar code, the failed reconstructed bar code, the corresponding clean convolved signal, the noisy convolved signal. The true parameter used to generate the corrupted signal: $M = 512$, $\sigma_0 = 0.0129$, $\alpha = 1$, $SNR = 16$. The estimated parameter: $\hat{\sigma}_0 = 0.0142$, $\hat{\alpha} = 1.067$. The square error $||\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}||_2^2/44 = 8.42e - 4$. CPU time: 44 seconds.

TABLE III

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\hat{\sigma}_0 - \sigma_0||_2^2)$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	1.161e-10	5.877e-08	1.697e-07
0.0120	512	5.333e-11	2.370e-09	1.600e-07
0.0120	1024	3.848e-11	3.171e-10	1.328e-07
0.0130	256	1.469e-07	1.897e-07	2.642e-07
0.0130	512	6.000e-08	7.565e-08	6.131e-07
0.0130	1024	7.824e-08	6.078e-08	3.015e-07
0.0133	256	8.639e-08	1.443e-07	3.862e-07
0.0133	512	1.909e-07	1.378e-07	5.560e-07
0.0133	1024	1.453e-07	1.378e-07	4.213e-07

in contrast the reported 6 minutes using the regularization approach in [9].

VI. CONCLUSION

A nonlinear least squares estimation for change points of bar code signals is proposed. The local information contained in the derivative of the convolved signal is used to provide starting values for the

TABLE IV

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\sigma_{0initial} - \sigma_0||_2^2)$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	1.064e-05	1.037e-05	1.528e-05
0.0120	512	2.481e-05	2.725e-05	3.486e-05
0.0120	1024	6.468e-05	6.560e-05	6.726e-05
0.0130	256	2.338e-05	2.063e-05	2.369e-05
0.0130	512	3.441e-05	3.587e-05	4.610e-05
0.0130	1024	8.140e-05	8.233e-05	8.489e-05
0.0133	256	2.808e-05	2.498e-05	2.757e-05
0.0133	512	3.717e-05	3.858e-05	4.635e-05
0.0133	1024	8.689e-05	8.797e-05	9.017e-05

TABLE V

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\hat{\alpha} - \alpha||_2^2)$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	4.138e-07	2.619e-04	7.075e-04
0.0120	512	2.330e-07	3.196e-06	9.139e-04
0.0120	1024	1.515e-07	1.154e-06	5.547e-04
0.0130	256	5.612e-04	7.304e-04	1.279e-03
0.0130	512	3.996e-04	4.341e-04	2.301e-03
0.0130	1024	5.613e-04	6.051e-04	1.617e-03
0.0133	256	3.699e-04	6.960e-04	1.844e-03
0.0133	512	9.908e-04	4.753e-04	2.429e-03
0.0133	1024	1.033e-03	1.221e-03	1.916e-03

global optimization solution. This hybrid approach uses all available information for parameter estimation to the full extent. Monte Carlo simulation results confirmed the good performance of the hybrid approach over the local approach.

If extra information such as the knowledge of the width of the thinnest or thickest black and white strips is available, they can be easily incorporated into the linear inequality constraints.

Currently, the value K of number of bars is assumed to be known in advance. A future research effort is to estimate the bar code without this assumption. Then model selection methods are needed for this

TABLE VI

MONTE CARLO APPROXIMATIONS TO $MSE = E(||\alpha_{initial} - \alpha||_2^2)$

		Means for the following levels of SNR		
σ_0	M	$SNR = 38$	$SNR = 28$	$SNR = 21$
0.0120	256	2.490e-02	2.464e-02	3.465e-02
0.0120	512	3.692e-02	4.138e-02	5.836e-02
0.0120	1024	8.754e-02	9.055e-02	9.720e-02
0.0130	256	3.904e-02	3.618e-02	4.055e-02
0.0130	512	5.158e-02	5.450e-02	6.901e-02
0.0130	1024	1.053e-01	1.075e-01	1.113e-01
0.0133	256	4.483e-02	4.049e-02	4.246e-02
0.0133	512	5.550e-02	5.592e-02	6.722e-02
0.0133	1024	1.104e-01	1.116e-01	1.133e-01

situation.

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